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V. ILICETO[1] and G. SANTARATO[2]

THE COMBINED INTERPRETATION OF VERTICAL ELECTRIC AND MAGNETOTELLURIC SOUNDINGS: A CASE HISTORY

Abstract. A case history in which magnetotelluric (MT) soundings and Vertical Electric Soundings (V.E.S.) were performed on a structurally simple geology is presented.

The comparison of MT synthetic apparent resistivity curves, computed using the one-dimensional model parameters derived from V.E.S. interpretation, with the experimental ones, is discussed. Inversion of MT data gives a range of oscillation for model parameters which is conditioned by data scattering. A careful assessment of the inversion effectiveness, by means of suitable statistical tests, allows us to single out the most statistically reliable model parameters.

INTRODUCTION

The application of the Magnetotelluric (MT) method in geophysical prospecting is based on the theory originally proposed by Cagniard (1953). Although MT on the whole offers several advantages with respect to other electrical methods, it is only marginally adopted by industry.

The difficulties for a routine use of MT can be divided into two categories:

1) problems related to the source of the MT field;

2) interpretational problems due to the complexity of some of the investigated structures.

In the first category, we find the difficulties related to the plane-wave approximation and the interaction of man-made noise with the natural signal.

In the second category, the problems range from the electrical equivalence and the assessment of 2D and 3D situations to current channelling and static shift (Jones, 1988).

Nevertheless, the present need to know the structure of the intermediate and deeper parts of the lithosphere - where data supplied by other geophysical methods are scarce or even lacking - has given a new impulse to the use of the MT method. In fact, MT enables one to investigate the electrical properties of the entire lithosphere by a simple extension of the period range, without resorting to artificial sources, which for great depths of investigation would require an extremely high power.

MT sounding data are affected by the same limits as all other electric and electromagnetic methods, i.e., the equivalence and suppression principles. Theoretical and model-based discussions of the limits imposed on both methods by the equivalence and suppression principles are widely reported in the literature (see e.g. textbooks such as Bhattacharyya and Patra, 1968 and Kaufman and Keller, 1981). Beyond these well known limits, the interpreter should also take into account the scattering of experimental data which is of considerable importance in MT data interpretation (see e.g. Rocroi, 1975).

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[1] Dipartimento di Geologia, Paleontologia e Geofisica, Università di Padova, Italy.

[2] Istituto di Mineralogia, Università di Ferrara, Italy.

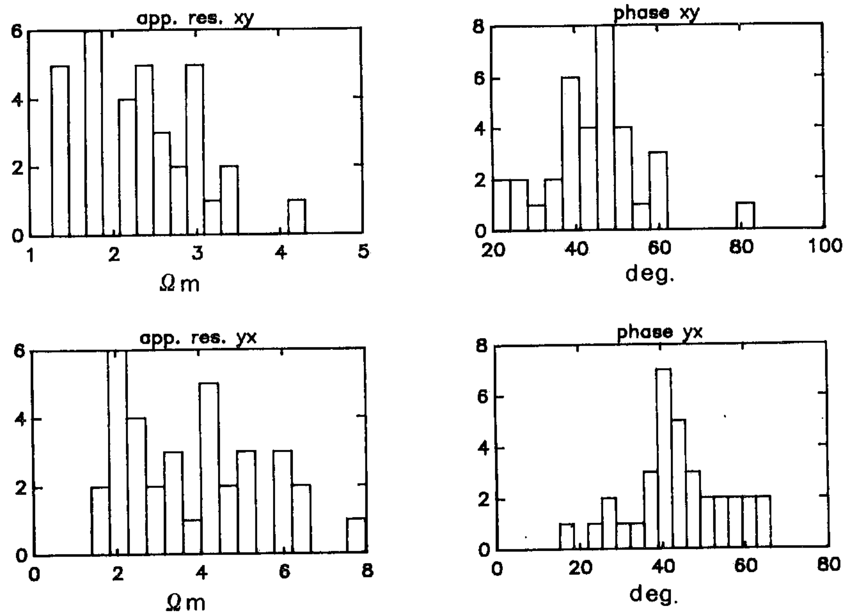

Fig. 1 — Bar histograms of apparent resistivity (left) and impedance phase (right) data from MT sounding no. 2, period 0.0528 s.

CONSIDERATIONS ON MT DATA SCATTERING

A closer examination should be devoted to this topic. Researchers were faced with the problem of data scattering right from the start of the MT method. The reasons for this drawback, which are inherent in the method itself, i.e. sources not at infinity, are well known, although their effect cannot be assessed. Very soon, as numerical data processing was adopted, suitable "validation" tests for field data were introduced. Among them, wide success was attained by the method of predicted coherency (Swift, 1967) between each measured electrical component $E_{i,\,m}$ $(i=x,\ y)$ and the same component $E_{i,\,c}$, computed from both magnetic components H_x H_y , weighted by their respective Z_{ij} $(j=x,\ y)$ element of the impedance tensor Z (i.e. $E_{i,\,c}=Z_{ix}\,H_x+Z_{iy}\,H_y$). These elements were computed by, for example, the linear least squares regression of FFT' ed time series (Sims et al., 1971). All values of apparent resistivity and impedance phase pertaining to frequency bands in which this test - and eventually others (see Jones et al., 1989) - is satisfied (predicted coherency greater than 0.9 or 0.95) are used to draw field curves by means of their average and their own error bar, which is generally equal to one standard deviation. The more or less implicitly accepted hypothesis for this presentation is that the mean is a good (i.e. unbiased) estimate of the true value and that fluctuations around the mean are of Gaussian nature. In fact, the gaussian or normal distribution is a very restrictive hypothesis, which is not satisfied by MT data. In fact, the χ^2 test for the normal distribution (or even log-normal distribution for apparent resistivity data) is often unsuccessful (see, e.g., Bentley, 1973), both because of data very far from the mean value (outliers) and because of strongly skewed or even multimodal distributions. In Fig. 1, we report as examples some histograms of apparent resistivity and phase data from MT sounding no. 2 (see below). The error bar should thus be considered as only a simplified and handy way to represent data scattering; the true value lies somewhere in its interior. A graphical presentation which better honours the described nature of MT data is thus the cloud of data points satisfying the validation tests. The practical consequence is that, as already stated by Rocroi (1975), any model, the curves of which remain inside the data clouds (or inside the error bars, provided these are considered in the above-discussed sense) is an equivalent solution of the inverse problem.

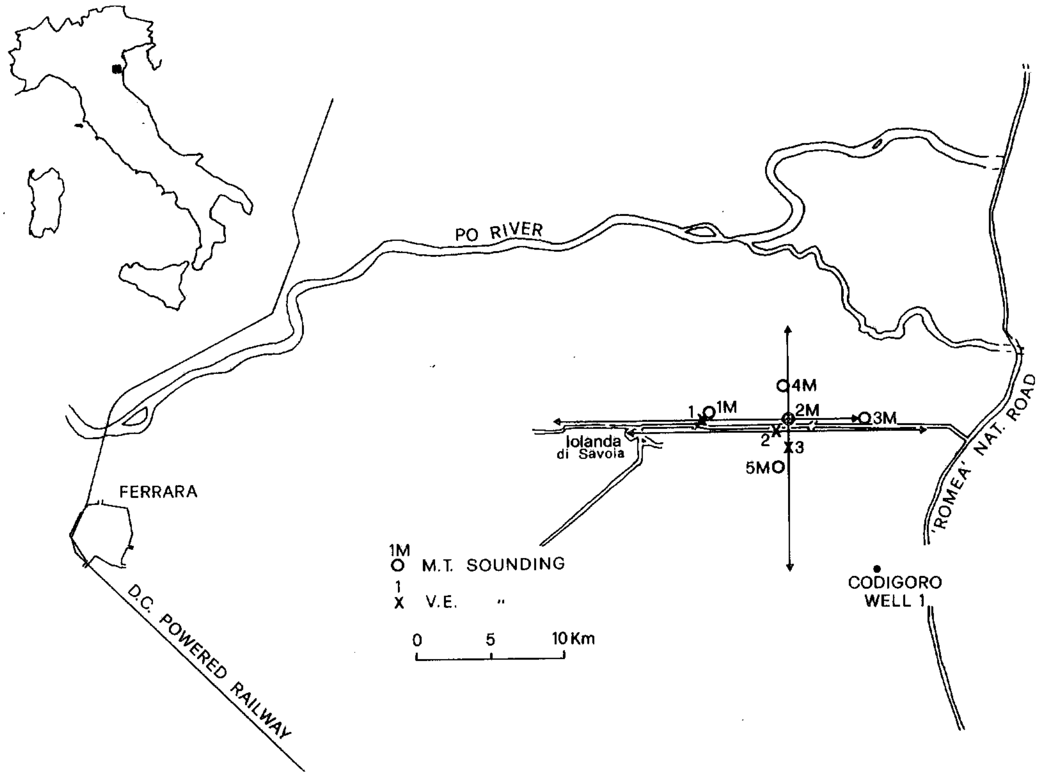

Fig. 2 — Measurements area.

Many analytical inversion schemes of MT 1D data have been introduced in the literature. Within them, suitable definitions of confidence limits and resolution of the estimated parameters are given. Among such tests, of particular significance are the error bounds (Inman, 1975; Jupp and Vozoff, 1975), defined as the oscillations of parameters around their best estimates (in the least squares sense). Such oscillations are computed on the basis of a maximum acceptable error, sized in its turn on the basis of data scatter. Also a test on parameter resolution is available as the parameter resolution matrix: for its definition see Inman et al. (1973). Each diagonal element of this matrix indicates how well the respective parameter (resistivity or thickness) is resolved by the dataset. Thus, if the matrix is the identity matrix, then all parameters are perfectly resolved; otherwise, it may indicate equivalence conditions.

Geological knowledge and/or geophysical data furnished by other geophysical methods (logs, d.c. soundings, seismic profiles) will give constraints for a choice among several possible, equivalent models.

Since MT soundings give apparent resistivity curves quite similar to those of vertical electric soundings (V.E.S.), a comparison of the respective limits and advantages is a very interesting topic. In this paper, we shall compare MT measurements with direct current (d.c.) Schlumberger V.E.S. carried out over a favourable, almost one-dimensional (1D) structure.

EXPERIMENTAL EXAMPLE

An area has been chosen in the eastern part of the province of Ferrara (northern Italy) not only for the simplicity of the buried structures there but also for the presumably low man-made noise level (Fig. 2).

The geology in this area, which belongs to the Po foreland, is well known on account of intensive geophysical surveys and numerous drillings carried out by the AGIP oil company for hydrocarbon exploration (see, e.g., Cassano et al., 1986). The structure is essentially tabular (Fig. 3) as it was formed by sedimentation in alternately marine and continental facies, and lies on the crystalline basement approximately 8 km deep, which gently rises towards the north. The outer Apennine front (Ferrara-Romagna folds) is situated several km to the west and south of the measurement sites, and thus from a geological point of view the buried structure can be considered as an extensive sequence of essentially horizontal layers.

The electrical behaviour of the subsoil had been defined by electrical logs taken in the AGIP wells, the nearest of which is the "Codigoro 1". The resistivity log of this well gives resistivity values for its entire length (well bottom 3512.2 m b.s.l.) less than some tens of Ω m (Cassano, AGIP, personal communication). This low resistivity pattern, together with the low industrialisation of the area as a whole, led us to suppose that the man-made noise level was quite low there.

On the basis of these considerations, this area was chosen for carrying out the MT test survey and for better defining the electrical properties of its subsoil deep, crossed Schlumberger-type V.E.S.'s were performed.

V.E.S. and their interpretation

Taking advantage of the area's favourable topographic conditions, 3 large-array Schlumberger-type V.E.S.'s were performed: V.E.S.'s nos. 1 and 2 with E-W azimuth and maximum AB distance of 20 km, and V.E.S. no. 3 with N-S azimuth and maximum AB distance of 16 km (see Fig. 2).

The field apparent resistivity curves are reported in Fig. 4. These are all characterised by the same H-sequence, except for slight differences in their shallowest part. Inversion - bearing in mind the data of the Codigoro 1 resistivity log - has supplied the best fitting 1D models as drawn on the right side of their respective field curves. The features of the apparent resistivity curves and especially the ones pertaining to crossed V.E.S.'s nos. 2 and 3 confirm the basically 1D nature of the buried structure.

MT soundings

In the neighborhood of the V.E.S. centres, five MT soundings were carried out, within the period range from 0.01 to 100 s, approximately. The apparent resistivity and phase curves were obtained by the method described in Iliceto and Santarato (1979). The curves were very similar in both measuring directions (reference x-axis along N-S and y-axis along E-W) through all MT soundings; accordingly the skewness s data, defined as

$$s = \left| \frac{Z_{xx} + Z_{yy}}{Z_{xy} - Z_{yx}} \right| ,$$

(Z_{ij}—$i, j = x, y$ — are the elements of the Z impedance tensor) were generally less than. 0.1 and the strike ∂ values, computed by the formula

$$\partial = 0.25 \tan^{-1} \frac{2 \, Real \, ((Z_{xx} - Z_{yy}) \, (Z_{xy} + Z_{yx})^*)}{|Z_{xx} - Z_{yy}|^2 - |Z_{xy} + Z_{yx}|^2}$$

(symbol * indicates the conjugate complex) showed no preferential strike direction. The investigated structure should thus be considered 1D, also from the MT point of view.

For the sake of brevity, we limit ourselves here to reporting (Fig. 5) the apparent resistivity curves yx (telluric component y, magnetic component x) for the MT soundings nos. 2 and 3, and xy (telluric component x) for the MT sounding no. 5; these curves refer to the same azimuths of V.E.S.'s nos. 1, 2 and 3, respectively.

The results are, as can be seen, affected by a unexpectedly high noise level, drawn as an error bar equal to one standard deviation around the mean (thus following the usual "Gaussian"

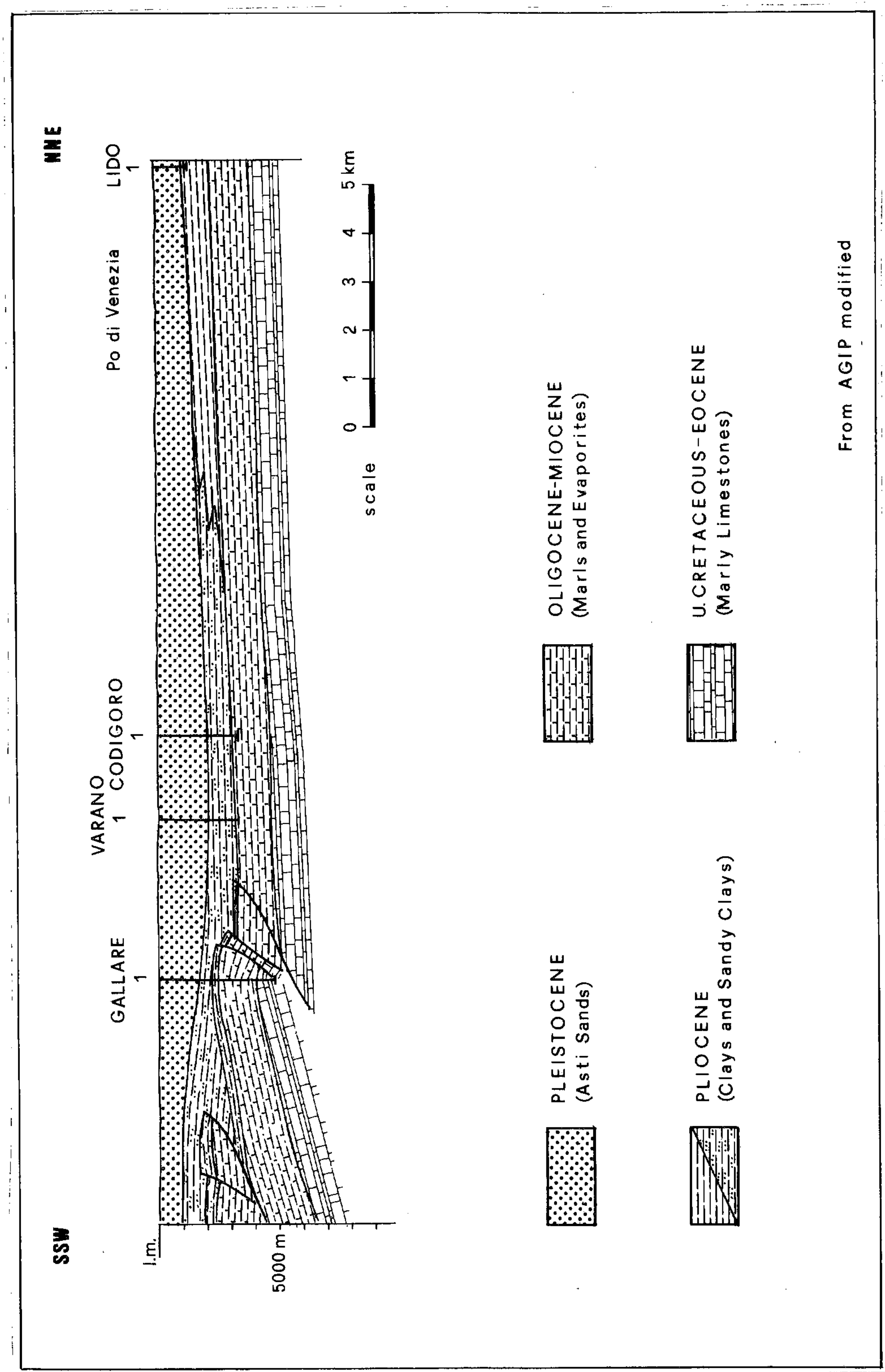

Fig. 3 — Schematic N-S geological section across the measurement area (modified from Cassano et al., 1986).

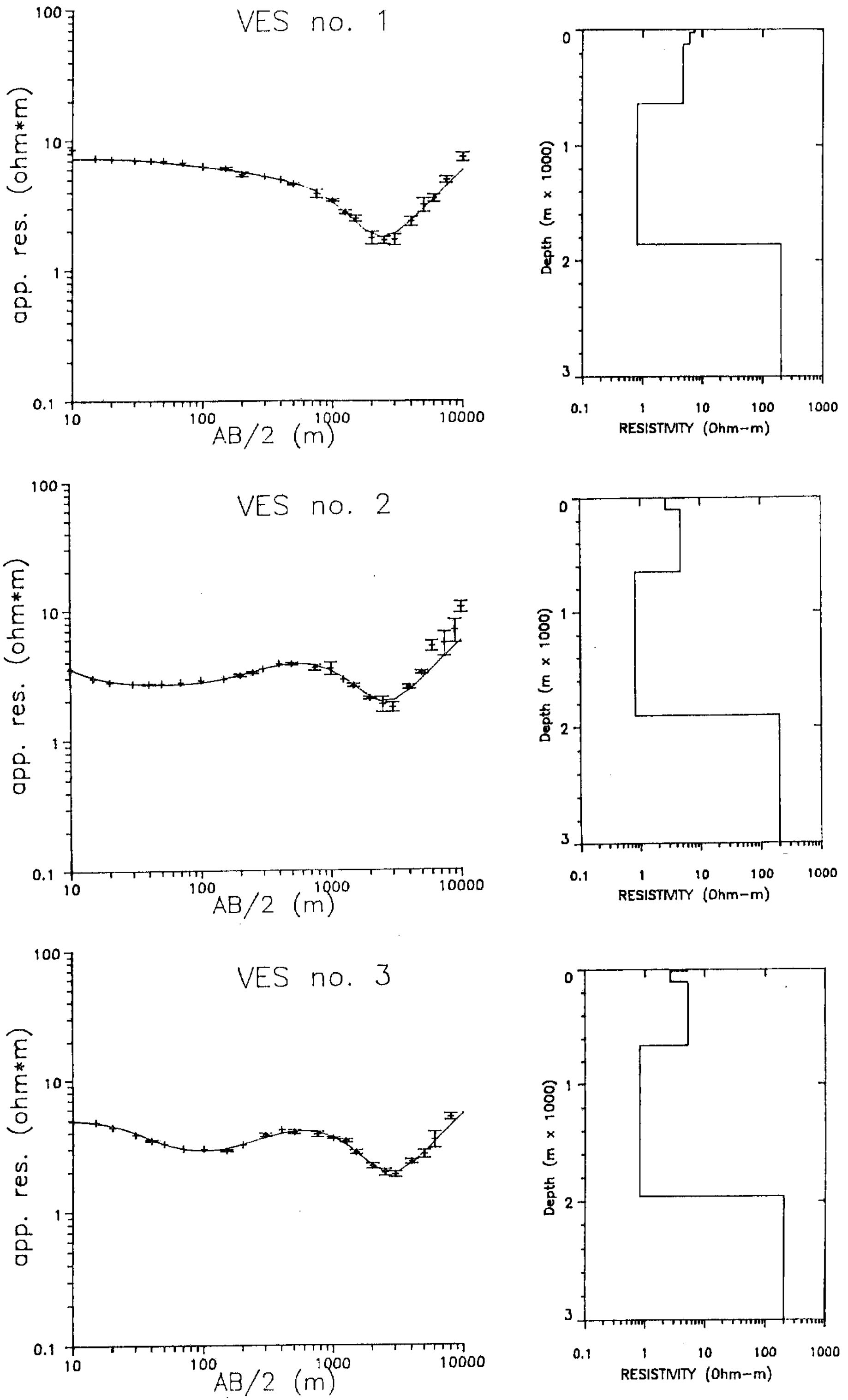

Fig. 4 — Schlumberger V.E.S. nos. 1, 2 and 3 apparent resistivity curves: experimental (asterisks and error bars where bars are greater than asterisks) and computed (continuous line) from the respective best fitting models drawn on the right.

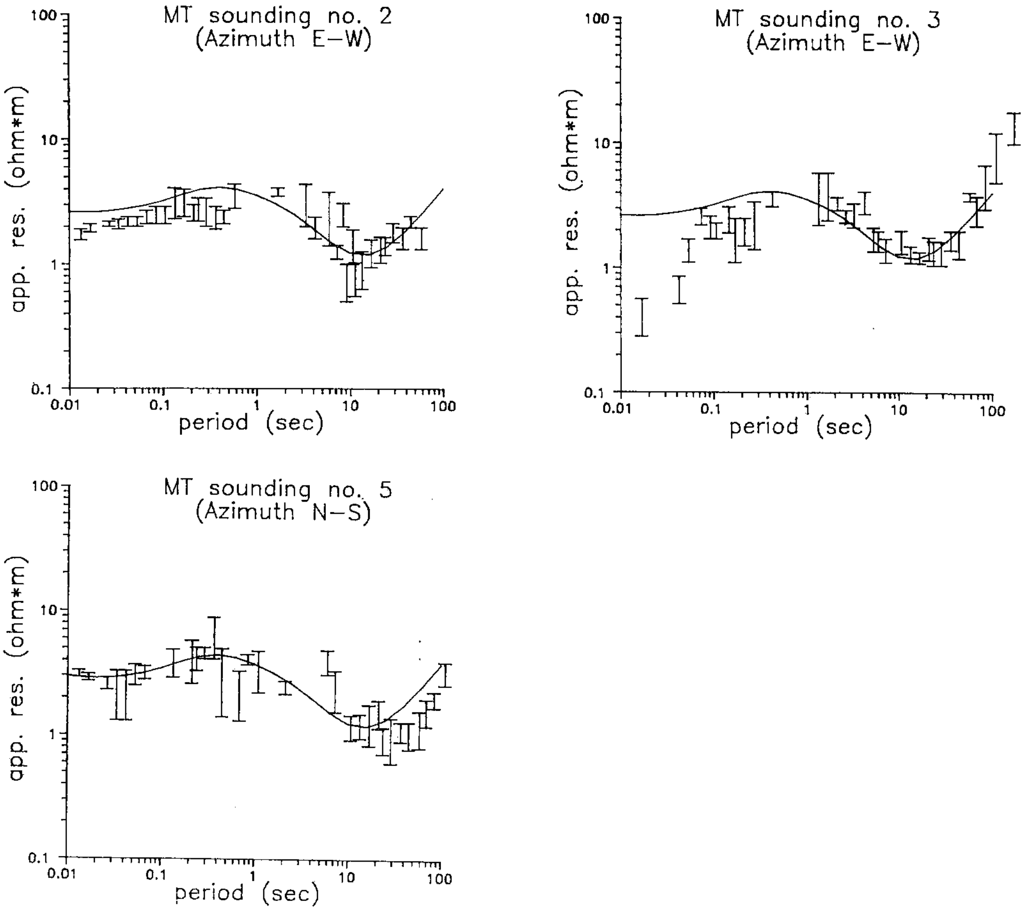

Fig. 5 — MT apparent resistivity and phase curves of soundings nos. 2, 3 and 5 (see text for azimuths): experimental (error bars represent one standard deviation around the mean value) and computed (continuous line) using the parameters of best fitting V.E.S. models nos. 2 and 3 respectively.

way for representing scattering); a similar noise level affected all other data not reported here. The purpose of this test was to verify whether and to what extent the inversion of MT data affected by scattering provides a realistic model of the subsoil.

First of all, we wished to check whether the true electrical information about the subsoil was present in the MT data. Therefore, the synthetic apparent resistivity curves for MT soundings nos. 2, 3 and 5 were calculated using the parameters of the best fit model of V.E.S.'s nos. 2 and 3, respectively. These curves are drawn as a continuous line over the MT field data. As expected, the MT "predicted" curves generally fall into the field data error bars, although with some misfit. A possible reason for this misfit may be a downward bias in the Z tensor elements estimates due to noise in the magnetic components.

Some authors (see e.g. Patella, 1987) suggest that discrepancies between d.c. and MT responses may be due to frequency dispersion of the electrical resistivity.

As a second step, an unconstrained inversion was carried out on the MT data alone, to analyze parameter reliability and stability. These were obtained both by computing the limits of the parameter ranges on the basis of the maximum acceptable error and by calculating the parameter resolution matrix. The maximum acceptable error was sized on the mean half-amplitude of the error bars. As an example, we report in the Table the results obtained by the inversion of MT site no. 2 (direction yx), compared with the same analysis carried out on the V.E.S. no. 2 data. The respective diagonal elements of the parameter resolution matrix are given in columns labeled "Res.".

Table — Detailed 1D inversion results of MT sounding no. 2 and V.E.S. no. 2.

		MT sounding no. 2 (max. rel. error: 20%)				V.E.S. no. 2 (max. rel. error: 5.5%)			
Layer		Minimum	Best	Maximum	Res.	Minimum	Best	Maximum	Res.
Resist. (Ohm.m)	1	1.47	4.65	5.39	.01	3.52	7.55	27.5	.29
	2	1.46	1.98	2.62	.85	2.47	2.66	2.83	1.0
	3	2.50	3.58	4.41	.63	3.95	4.72	6.12	.97
	4	0.34	0.86	1.35	.69	0.36	0.71	1.16	.55
	5	9.19	91.9	919.	.02	20.9	310.	3100.	.01
Thick. (m)	1	0.99	13.5	34.9	.04	1.59	2.67	3.99	.85
	2	18.9	94.8	155.	.20	72.2	108.	165.	.88
	3	555.	748.	1344.	.65	338.	522.	702.	.88
	4	439.	1453.	2611.	.59	560.	1180.	1910.	.52

(Res. = diagonal element of parameter resolution matrix)

Obviously, due to the much greater scatter in the MT data, the parameters given by V.E.S. data inversion are far more reliable.

But note that resistivity values of layers nos. 2, 3 and 4 are well enough resolved in the MT inversion to save the fundamental KH sequence feature. Remember that for this test no model parameter was kept fixed, as we could do, by taking into account all other electrical data. Note also that the resolution for parameters of layer no. 4 is better in MT than in V.E.S. data: this means that resolution of this layer is more hindered by equivalence in V.E.S. than in MT data.

Finally, the presence in these MT and V.E.S. data of a final branch rising at limit slope (+1 in log-log coordinates) allows for an assessment of the total conductance of the sedimentary layers above the resistive substratum; this last datum is well determined both from graphical extrapolation and from the parameters of the MT and V.E.S. best fitting models (columns "Best" of the Table).

CONCLUDING REMARKS

The present work was aimed at assessing in a real case the reliability of the information given by MT data, when noise is also present.

A geologically simple site was thus chosen, with the goal of avoiding in this experiment any interpretative problems due to the complexity of the buried test structure. The electrical response of the site was then completed by carrying out three deep, crossed Schlumberger V.E.S.'s.

MT results were affected by a notable noise, in spite of both remoteness of the probable noise sources (electrified railways, industrial plants) and low resistivity of the thick alluvial mattress in the test area. Nevertheless, the MT data were consistent with the electrical features of the investigated structure and their intentionally unconstrained inversion gave enough reliable basic information, such as number and sequence of layers.

True resistivities of intermediate layers above the basement, together with the total conductance of the conductive sequence, were also resolved.

As reliability of inversion is heavily affected by data scattering, an essential step in the MT method should be a pass through a data processing scheme which, starting from non-restrictive hypotheses about MT data statistics, would reduce scattering, whenever intermediate to high noise is present.

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