

Time series pre-processing techniques for improving the fundamental frequency assessment in H/V seismic spectral ratios

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ABSTRACT In the context of seismic site effect characterisation, the *H/V* technique is able to identify efficiently the fundamental frequency of a soft surface layer upon a bedrock. Compared to other techniques for such experimental investigations, the *H/V* approach is one of the cheapest and therefore most widely used. The direct application of the *H/V* spectral ratio, however, provides results that are not always easy to analyse; for this purpose, it can be extremely useful to look for pre-processing algorithms that help highlight the characteristics of the signal we are most interested in, in particular, the position of the peak associated to the fundamental frequency. In this paper we summarise the main lines of research developed by our research group at the “Università degli Studi di Udine”, i.e. the Singular Spectrum Analysis (SSA) and the wavelet analysis (WASEE). A particular focus is put on the application of these techniques in combination with a previous rotation of the coordinate system in order to maximise the efficiency provided by the SSA and/or the WASEE.

1. Introduction

The effect of an earthquake in terms of damage can show strong geographical variations, even at short distances. This phenomenon is known as “site effect”. In particular, the possibility of estimating the amplification in given frequency ranges of the incoming seismic waves has important practical implications in terms of the correct estimation, and possibly minimisation, of the seismic risk. Several techniques are available for such experimental investigations, that largely differ in terms of costs and reliability of results. One of the cheapest and, therefore, most widely used methods is the so called *H/V* technique (Nakamura, 1989, 2000), that we will shortly describe. It is based on the spectral ratio between horizontal and vertical components of the continuously recorded ground motion. This ground motion is often called seismic noise, tremor or micro-tremor. We will use the word “tremor” in the rest of this paper, without associating any specific hypothesis in terms of source to this term. Although there is a consistent amount of research investigating the wavefield of this tremor [see e.g. the recent European research, program SESAME (2004), Lachet and Bard (1994), Arai and Tokimatsu (1998, 2000), Konno and Ohmachi (1998)], this is not relevant to the analysis we present in this paper, as no hypothesis on the wavefield is required for the algorithms to be applied here.

Usually, the tremor samples are obtained by measuring two horizontal directions (typically east-west and north-south) and a vertical one. This fact allows us to compute two independent

spectral ratios and compare them (Carniel *et al.*, 2006, 2007). If we suppose that the noise is generated by random and far sources and the site is characterised by a homogeneous and elastic surface layer medium, with planar horizontal layers, then the two ratios should be almost equal; in reality, due to non-ideal (i.e. real) conditions, the two ratios may be in disagreement, and they often show this in practice.

In the last few years, a line of research has been developed at the University of Udine, aimed at reducing this difference by preprocessing techniques such as the Singular Spectrum Analysis [SSA: Carniel *et al.* (2006)] or the wavelet analysis [WASEE: Carniel *et al.* (2007)]. Both these techniques were introduced for the case of laterally homogeneous layers, and random and far sources, but in the presence of laterally non-homogeneous layers or directional or non-white noise sources these techniques can fail, and/or their degree of success depends on the orientation of the (source and site) anisotropies with respect to the direction of the components recorded by the seismometers.

Therefore the following step was to remove this limitation by performing all possible rotations of the coordinate system and recomputing the spectral ratio in each new system. In this way we define the directions of maximum and minimum matching between the ratios. The direction of minimum matching refers to the maximum anisotropy and viceversa. If needed (the rotation can sometimes provide sufficient results on its own), the SSA or WASEE techniques can be applied more efficiently after the rotation procedures. In the following, we summarise these three approaches and show examples of applications.

2. Theoretical fundamental frequency

As described in Carniel *et al.* (2006) let's consider a sedimentary layer upon a bedrock. The sedimentary layer behaves like a filter, amplifying some frequencies and attenuating others. The frequencies for which the maximum amplifications are observed are called resonance frequencies and their knowledge can provide important information about the sedimentary layer. If the material is linear elastic (hypothesis which is generally acceptable for small deformations) and homogeneous, the resonance frequencies are given by the quarter wave law:

$$f_{shear, k} = (2k + 1) \frac{c_s}{4H} \quad (1)$$

$$f_{shear, k} = (2k + 1) \frac{c_s}{4H} \quad (2)$$

where c_s and c_p are respectively the propagation velocity of the S and P waves in the superficial layer and H its thickness.

We can associate a corresponding Eigen-mode (or Eigen-shape) to each Eigen-frequency; as an example, the first three Eigen-modes are represented in Fig. 1.

For engineering purposes it is more important to know $f_{shear, k}$ than $f_{long, k}$ because buildings

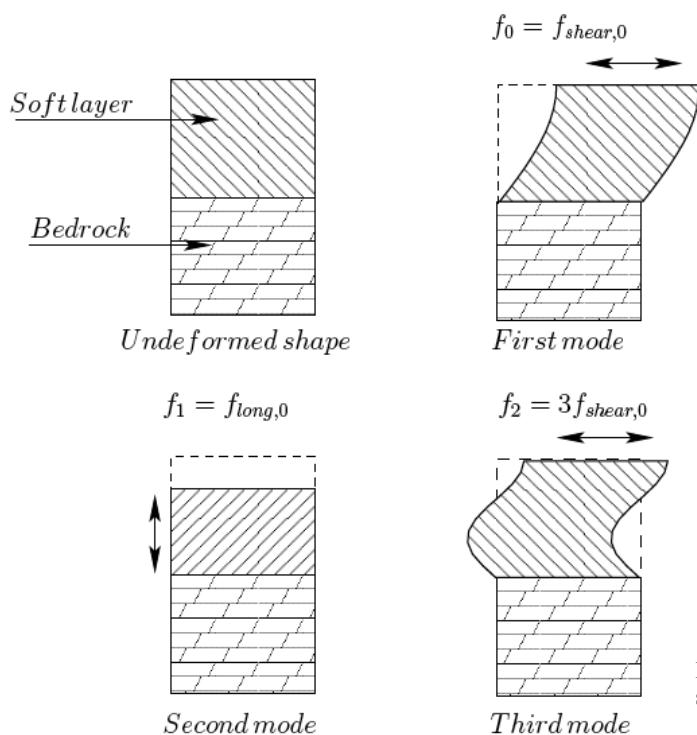


Fig. 1 - The first three Eigen shapes in a soft surface layer over a bedrock.

are more resistant to vertical forces/accelerations than to the horizontal ones. For this reason, we concentrate on the fundamental frequency (f_0) of the shear mode. An obvious way of using this information is to design buildings attempting to avoid that their fundamental frequencies correspond to the ones of the surface layer on which they are to be built.

The fundamental frequency f_0 is obtained for $k=0$, so

$$f_0 = f_{shear, 0} = \frac{c_s}{4H}. \quad (3)$$

3. Basics of H/V technique

The technique was first proposed by Nakamura (1989) to estimate the dynamical characteristics of surface layers by measuring, solely, the tremor at the surface. Fig. 2 depicts the typical geological structure of a sedimentary basin. Here, the tremor is divided into two parts, assuming it contains surface and body waves. The latter will be modified by the filtering action of the soft layer. It is therefore possible to define the spectra of the horizontal (H_f) and vertical (V_f) surface motion as:

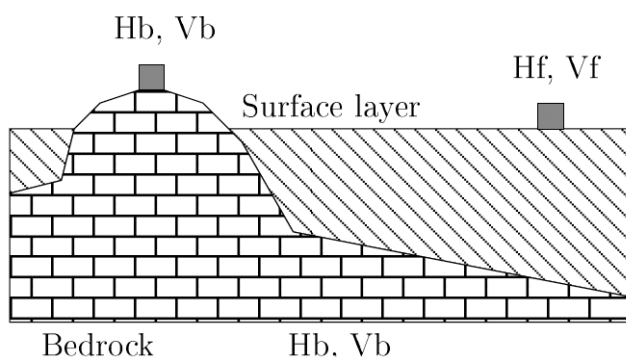


Fig. 2 - Typical geological structure of a sedimentary basin.

$$\begin{cases} H_f = A_h \cdot H_b + H_s \\ V_f = A_v \cdot V_b + V_s \end{cases} \quad (4)$$

where A_h and A_v are the amplification factors of the horizontal and vertical motion of vertically incident body waves, while H_b and V_b are the spectra of horizontal and vertical motion at the basement under the basin. H_s and V_s are the spectra of horizontal and vertical directions of surface waves.

Nakamura (1989) considered the following ratio:

$$\frac{H}{V} = QTS = \frac{H_f}{V_f} = \frac{H_b}{V_b} \cdot \frac{A_h + \frac{H_s}{H_b}}{A_v + \frac{V_s}{V_b}} \quad (5)$$

where *QTS* is the acronym for Quasi Transfer Spectra.

Having the ideal configuration described above, the ratio should show a peak in correspondence to the fundamental frequency f_0 . More generally, for low frequencies, the H/V ratio is assumed to be a good approximation of the transfer function of the surface layer. This offers a great logistical advantage over the classical “reference site” approach.

It is noteworthy that although this method is widely applied, the original hypotheses made by Nakamura (1989) on the wavefield are highly questioned nowadays (see e.g. SESAME, 2004; Arai and Tokimatsu, 1998, 2000; Konno and Ohmachi, 1998; Lachet and Bard, 1994), but we stress that none of these hypotheses are needed or used by the time series analysis techniques we describe in the following.

4. Review of the pre-processing techniques

In this section, we present a short review of the main lines of research of the group at the

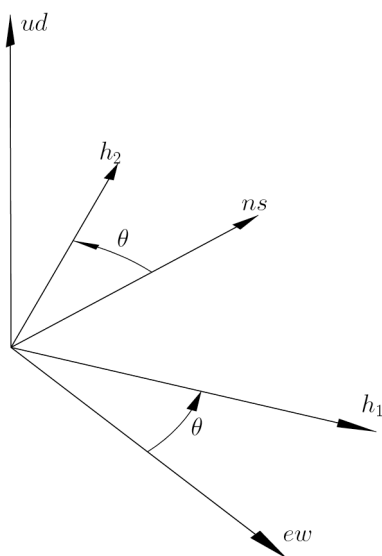


Fig. 3 - The new coordinate system is obtained by a rotation of the horizontal components by an angle θ .

rotation of a suitable angle θ (Fig. 3), as proposed by Barazza *et al.* (2008). In this new system the components of the signal become:

$$\begin{cases} h_1 = ew \cdot \cos \theta + ns \cdot \sin \theta \\ h_2 = -ew \cdot \sin \theta + ns \cdot \cos \theta \\ v = ud \end{cases} \quad (6)$$

If $S(\bullet)$ represents the spectral amplitude of each component, then the H/V ratio becomes:

$$\begin{cases} QTS_1(f, \theta) = \frac{S(h_1)}{S(v)} \\ QTS_2(f, \theta) = \frac{S(h_2)}{S(v)} \end{cases} \quad (7)$$

In order to find the optimal value of θ we study the coupling between QTS_1 and QTS_2 . In this way we get two angles: θ_{max} and θ_{min} , that respectively maximise and minimise the coupling. The angle θ_{max} determines the two orthogonal horizontal directions along which the site behaves in the most isotropic way. Viceversa we can associate θ_{min} to the two orthogonal directions with the most anisotropic behaviour.

The coupling between the two ratios can be computed in several ways. The simplest one is to

University of Udine. We then focus on the rotation of the coordinate system and the improvements it can provide when applied in conjunction with SSA and wavelet techniques.

4.1. Review of the rotation of the coordinate system

The tremor is usually measured along the two horizontal directions east-west (*ew*) and north-south (*ns*) and the vertical direction (up-down, *ud*). If we consider the ideal configuration in which the sources are random and far from our site and the layers are effectively horizontal, parallel, homogeneous and elastic, the two ratios should be the same. In the real case, due to non-ideal source or site conditions, the two ratios often disagree.

If we suppose that the sources are not far from ideal, we can try to minimise the effects of the presence of non-horizontal and inhomogeneous layers by changing the coordinate system by a

use the least square (Euclidean distance) so that θ_{max} satisfies:

$$\|QTS_1(f, \theta_{max}) - QTS_2(f, \theta_{max})\|_{f \in I} = \min \quad (8)$$

and θ_{min} :

$$\|QTS_1(f, \theta_{min}) - QTS_2(f, \theta_{min})\|_{f \in I} = \max \quad (9)$$

To improve the evaluation of the fundamental frequency, the distance between the two QTS 's is computed only in a suitable sub-range around a reasonable a priori estimate of the fundamental frequency. This definition of distance is later named partial distance.

Also, the correlation function as a function of the rotation angle θ is investigated. For every θ the correlation between the two QTS 's [calculated as in Eq. (7)] is computed; in this way, it is possible to construct the behaviour of the correlation as a function of θ . This function can help to understand if the coupling between the QTS 's is only due to chance.

In order to provide a statistical support for our considerations some random signals have been generated: 3000 random "tremor measurements" (one measurement consists of three components signals) are generated and investigated. We note that, for a random signal, the correlation between QTS_1 and QTS_2 does not depend (in a statistical sense) on the rotation angle θ , showing a constant mean μ of about 0.5 and a standard deviation σ of about 0.079, with an almost Gaussian behaviour. So we can assume that if two signals show a correlation greater than $\mu + 1.96\sigma \approx 0.65$ or lower than $\mu - 1.96\sigma \approx 0.34$ then the probability for their "spectral coupling" being random is lower than 5%. In Fig. 4 the confidence interval for the correlation between random signals is shown.

We now go back to our experimental tremor measurements, for which the standard H/V ratio is computed for each rotation angle from 0° to 90° . The "partial distance", the distance and the correlation between the ratios as a function of the rotation angle are then computed. Looking at the values of the correlation it is possible to decide if the spectral matching between the two signals is random or not. Afterwards, the values of the partial distance are used to estimate the directions of maximum and minimum matching. For the data we analyse, the partial distance is evaluated using the interval $I=[2,10]$ Hz.

For comparison, an example of partial distance and correlation between QTS_1 and QTS_2 is shown in Fig. 5 for a random signal as a function of the rotation angle θ . The correlation is always contained in the confidence interval shown in Fig. 4.

4.2. Short review of SSA

The SSA technique allows us to decompose the signal into a sum of functions of decreasing importance. It consists of several steps (Golyandina *et al.*, 2001; Aldrich and Barkhuizen, 2003). In the first step (embedding), the one-dimensional time series is recast as an m -dimensional time series (trajectory matrix). In the second step (singular value decomposition), the trajectory matrix is decomposed into a sum of orthogonal matrices of rank one. These two

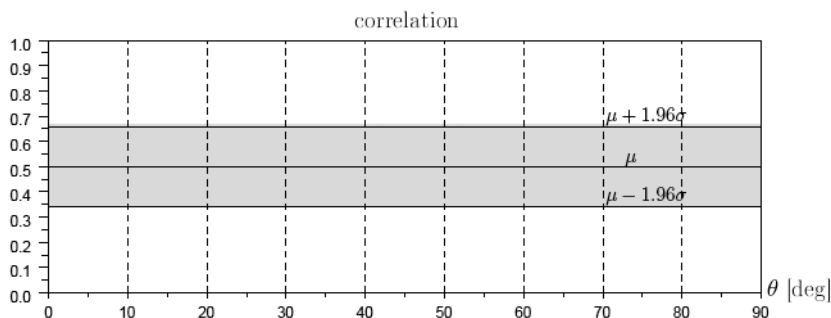


Fig. 4 - Confidence interval of 5% for the correlation between and for a random signal, as a function of the rotation angle θ .

steps constitute the decomposition stage of SSA. In the third and fourth steps, the components are grouped and the time series associated with the groups are reconstructed (Fig. 6):

$$s(t) = \sum_{i=1}^m pc_i(t).$$

In this way one can decompose the time series as

$$s(t) = s_1(t; m, q) + n(t; m, q)$$

where

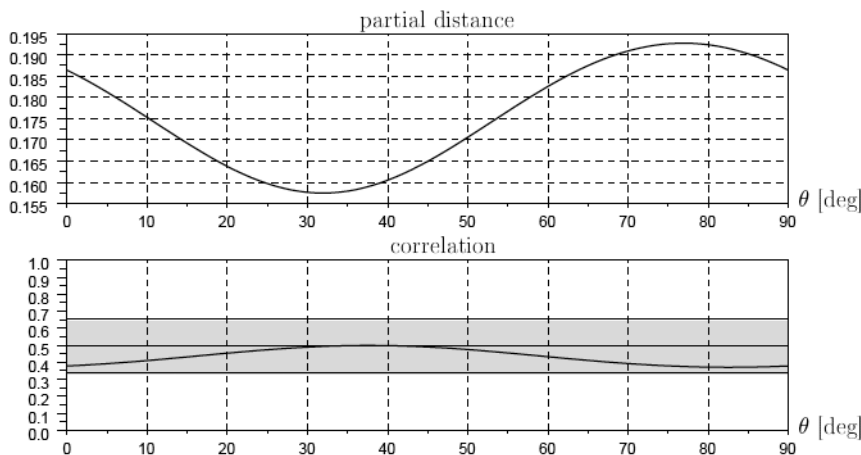


Fig. 5 - Example of partial distance, and correlation between QTS_1 and QTS_2 for a random signal as a function of the rotation angle θ .

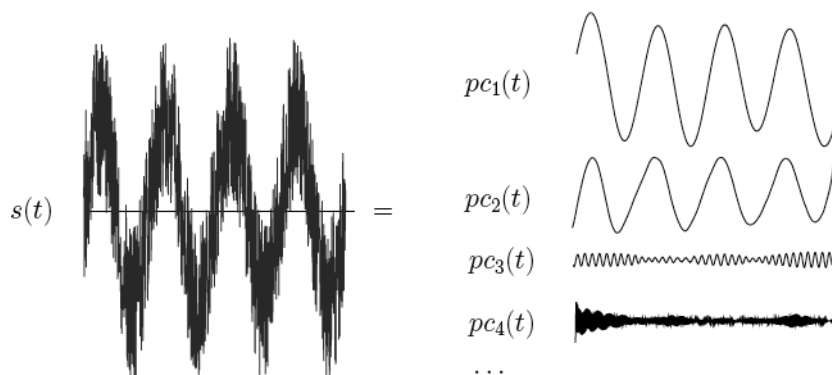


Fig. 6 - Example of a decomposition of a signal by the SSA technique.

$$s_1(t; m, q) = \sum_{i=1}^q pc_i(t)$$

is a hypothetical noise-free signal and

$$n(t; m, q) = \sum_{i=q+1}^m pc_i(t)$$

is the noise, according to the general definition of “what we are not interested in”.

Given the embedding dimension, one can then obtain m different levels of de-noising of the signal, from the strongest, $q=1$, to the weakest $q=m$ (the original signal itself).

The main idea of the application of the SSA to the H/V technique, as described in Carniel *et al.* (2006), is to choose the de-noising level of the measured tremor such that it maximises the matching of the spectral ratios between two orthogonal horizontal directions.

4.3. Short review of WASEE

The acronym WASEE means Wavelet Analysis for Site Effects Estimation. The algorithm is presented in detail in Carniel *et al.* (2007). The analysis can be summarised as follows (see also Fig. 8):

- the analysis of the signal is done by using a series of filter processes (as in Fig. 7) through a low-pass filter ($G(\bullet)$) and a high-pass filter ($H(\bullet)$) defined by the mother wavelet. The signal is then decomposed for each level, as the sum of different nodes, that are computed by filtering the nodes of the previous level. The first node (at the top of the figure) corresponds to the whole signal. In the figure, the term w refers to the wavelet coefficients of the signal; the first index indicates the level, while the second one refers to the node number. The Wavelet Packet Transform (WPT) of ew and ns signals is computed by using a symlet wavelet;

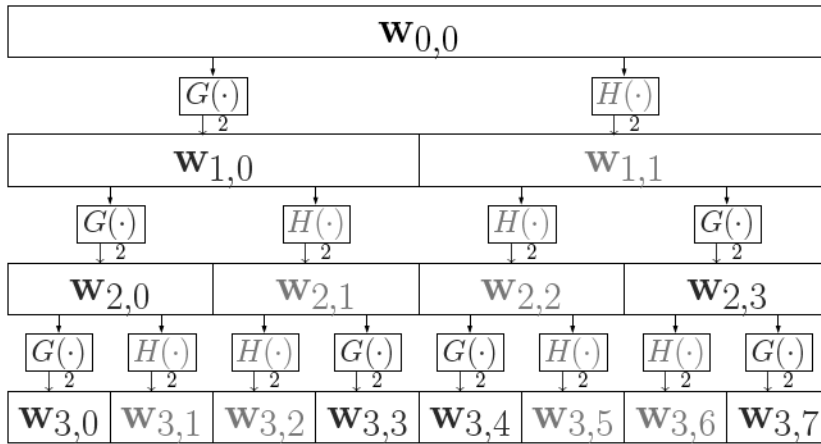


Fig. 7 - Schematic representation of the wavelet packet transform [modified from Percival and Walden (2000)].

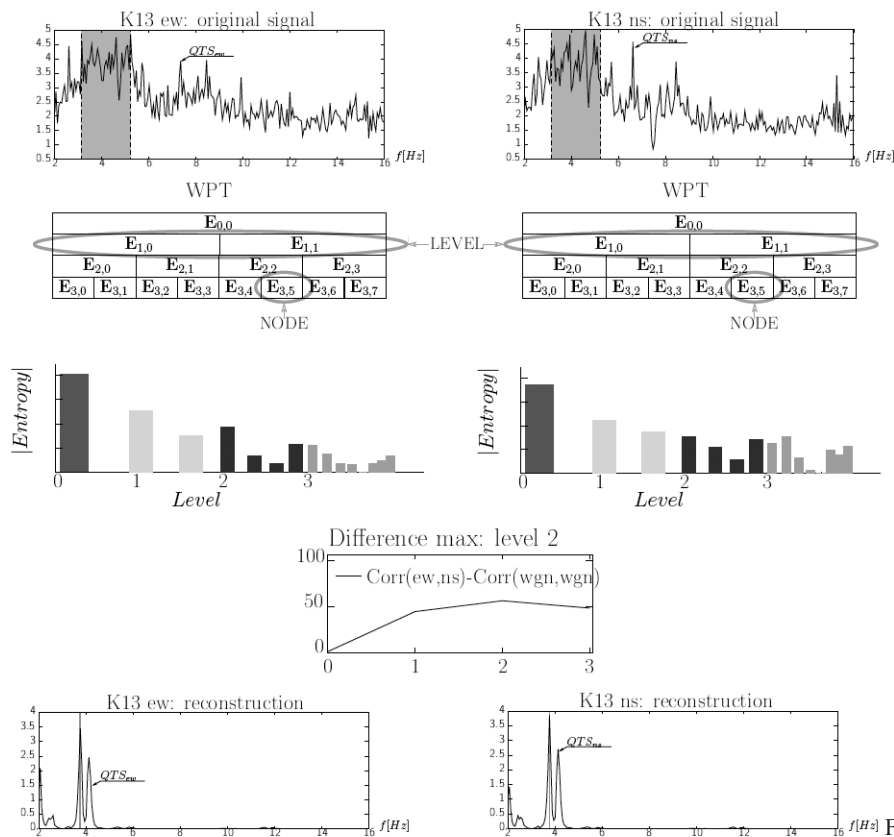


Fig. 8 - Synthesis of WASEE algorithm.

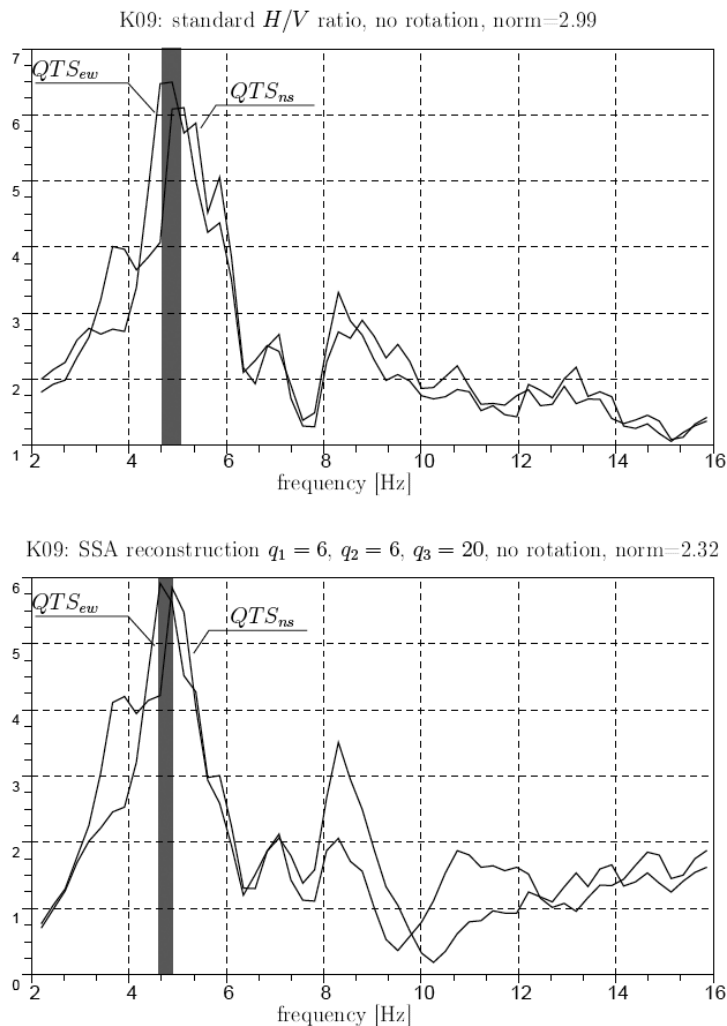


Fig. 9 - Standard H/V ratio and SSA ratio for the K09 station, before the rotation of the coordinate system.

- the value of Shannon’s Entropy, as defined in Coifman and Wickerhauser (1992), for each node of the decomposition is computed;
- the working level is chosen by evaluating the correlation between the two coefficient signals (this correlation should be as different as possible with respect to the correlation of two random signals);
- in the selected level the nodes that present the highest value of the entropy will be added until the peak of the new signal is sufficiently energetic (compared to the original one).

5. A test case

In order to verify the results, the same experimental signals described in Carniel *et al.* (2006) and Carniel *et al.* (2007) have been studied.

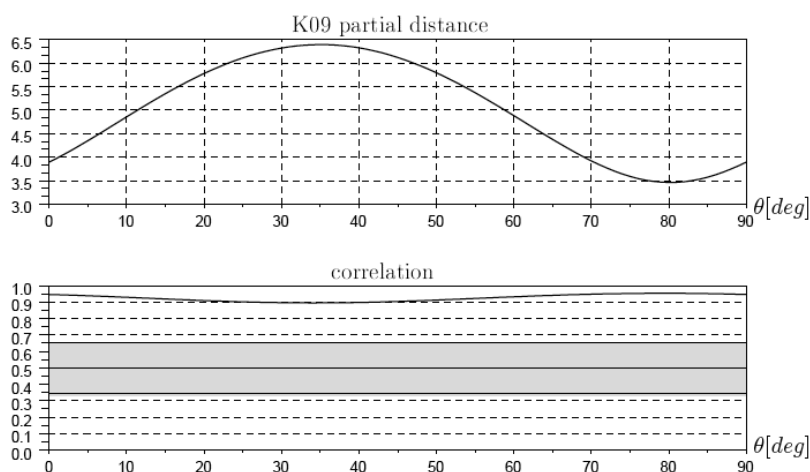


Fig. 10 - Partial distance and correlation between QTS_1 and QTS_2 for the K09 station as a function of the rotation angle θ .

The tremor measurements were recorded at Erba di Tarcento, a small village near Udine (Friuli, Italy) together with a seismic refraction survey. Each tremor sample contains about 56,000 data points (about 15 min at 62.5 Hz with a 1 Hz Lennartz LE-3D seismometer) for each station point along the seismic profile. For all tremor measurements, we first remove any trend from the signal and then we compute the spectrogram for each of the $2^8 = 256$, 50% points, overlapping Hanning windows (this allows a frequency resolution of about $f_{sample}/2^8 = 62.5/256 \approx 0.25$ Hz). In this way we can investigate the spectral stability and, if necessary, remove the portions of signals that are clearly contaminated by external (mainly anthropogenic) noise by using classic techniques like STA/LTA and removing the first and the last samples of the signal. Next, we perform a high-pass filter at 2 Hz on the whole signal, as the reasonable frequency range for f_0 is in this case between 2 Hz and 16 Hz, due to the presence of a rather thin superficial layer we want to characterise.

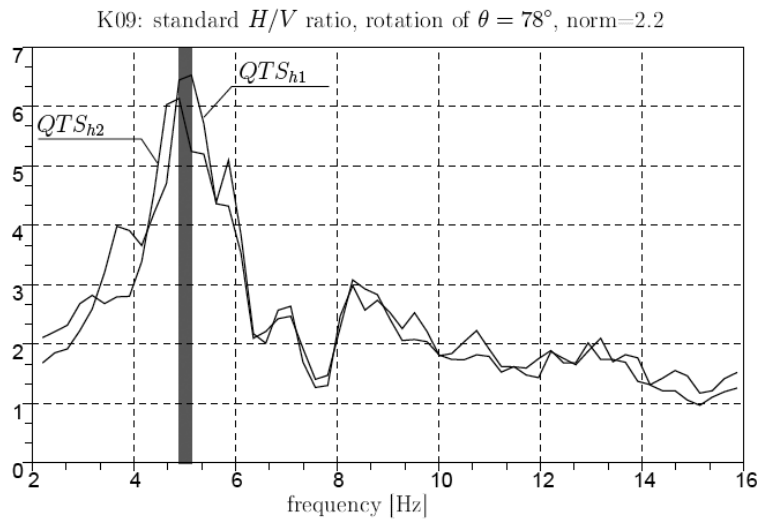
We consider a full seismic profile of 21 station points K02, K03, ..., K22 deployed at Erba.

In some cases the simple rotation of the coordinate system already allows a clear detection of the fundamental frequency. In this case the application of the SSA or WASEE technique is not necessary. In other cases, the rotation of the coordinate system does not yet provide a clear identification of the peak; then the SSA or WASEE can be applied in an optimal way, i.e. using the two horizontal components that already guarantee the maximum isotropy.

In Fig. 9, the standard H/V ratio and the SSA ratio are shown before the rotation of the coordinate system for the K09 station. In this case, the fundamental frequency f_0 can be determined with an uncertainty that the application of the SSA allows us to decrease.

In Fig. 10, the behaviour of the “partial distance” and the correlation between the QTS ’s is shown. From the investigation of the autocorrelation function one can verify that the degree of coupling between the QTS ’s is not random with a probability greater than 95%. The analysis of the “partial distance” then shows that the angle that minimises the distance between the QTS ’s (in the interested frequency range I) is $\theta = 78^\circ$.

Fig. 11 shows the standard H/V ratio and the SSA ratio after the rotation of the coordinate



K09: SSA reconstruction $q_1 = 8$ $q_2 = 8$ $q_3 = 20$, rotation of $\theta = 78^\circ$, norm=1.81

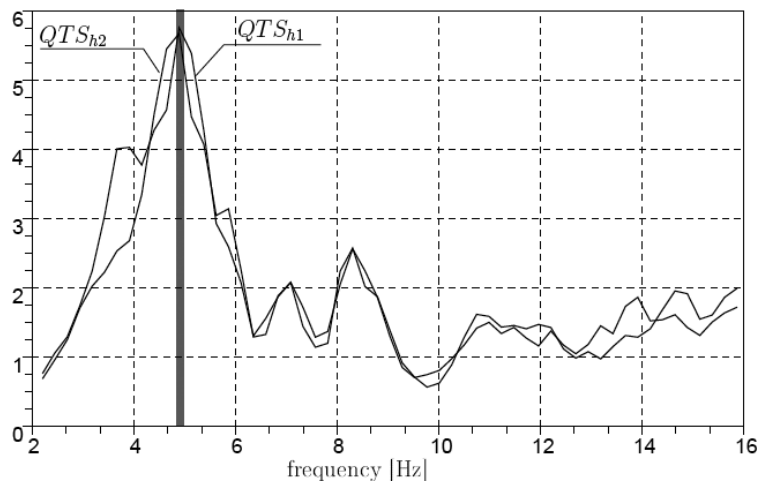


Fig. 11 - Standard H/V ratio and SSA ratio for the K09 station, after the rotation of the coordinate system by an angle $\theta = 78^\circ$.

system by the angle $\theta = 78^\circ$. The estimate of the fundamental frequency f_0 is improved with respect to the values obtained by Carniel *et al.* (2006), and better matches the estimate given by refraction seismics.

Also the WASEE technique can work better after the rotation of the coordinate system. Fig. 12 shows the standard H/V ratio and the reconstruction by WASEE for the station K11 before the rotation. The standard ratio shows two peaks defining quite a large band of frequencies for f_0 , while the application of WASEE [as the SSA in Carniel *et al.* (2006)] gives a narrow band for f_0 .

Fig. 13 again shows the standard H/V ratio and the reconstruction by WASEE, but only after the rotation of the coordinate system by an optimal angle of $\theta = 61^\circ$. The standard ratio shows only a little improvement in the determination of the fundamental frequency f_0 , but the

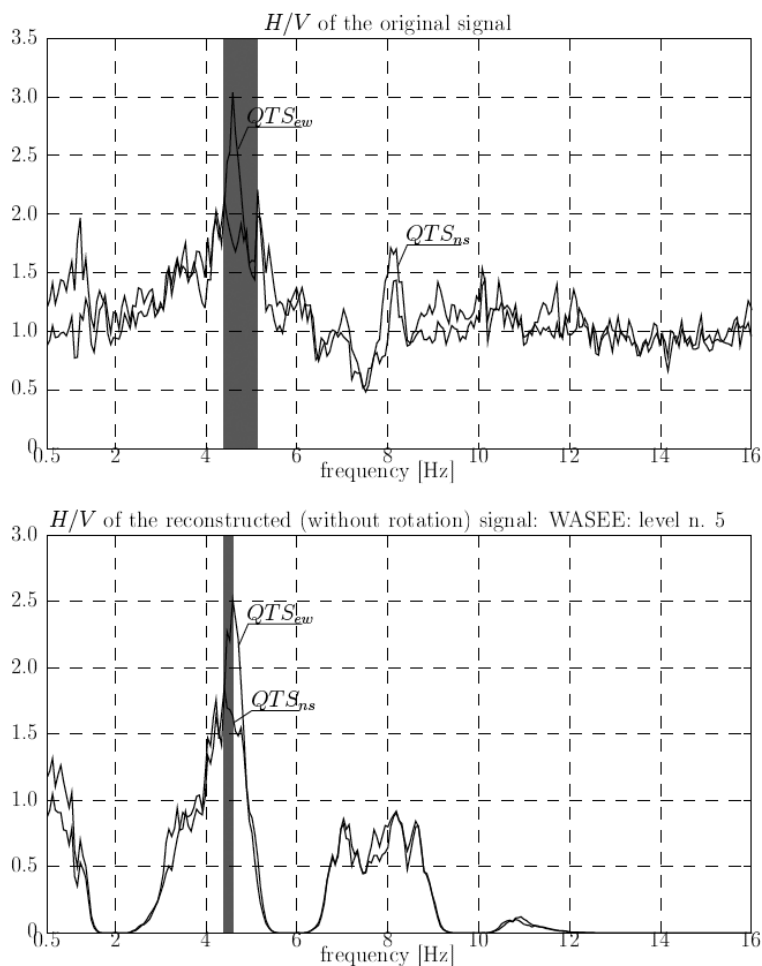


Fig. 12 - Standard H/V ratio and WASEE ratio for the K11 station, before the rotation of the coordinate system.

subsequent application of the WASEE reconstruction clearly shows a single peak, this time at about 4.8 Hz, a value that is in better agreement with the refraction seismic results [see also Fig. 13 of Carniel *et al.* (2006)].

6. Conclusions

It is widely accepted by the seismological community that the H/V technique does not give a systematic result. Nevertheless, when a peak is noticeable on the H/V ratios, the fundamental Eigen-frequency estimated is reasonably reliable, although the associated amplitude does not provide any consistent information. Therefore, the development of new pre-processing techniques that help to highlight an Eigen-frequency when this is not evident using the raw data, is of the uttermost importance.

The directions along which tremor measurements are made (typically north-south and east-west) are not related either to the real morphology of the site under investigation or to the possible

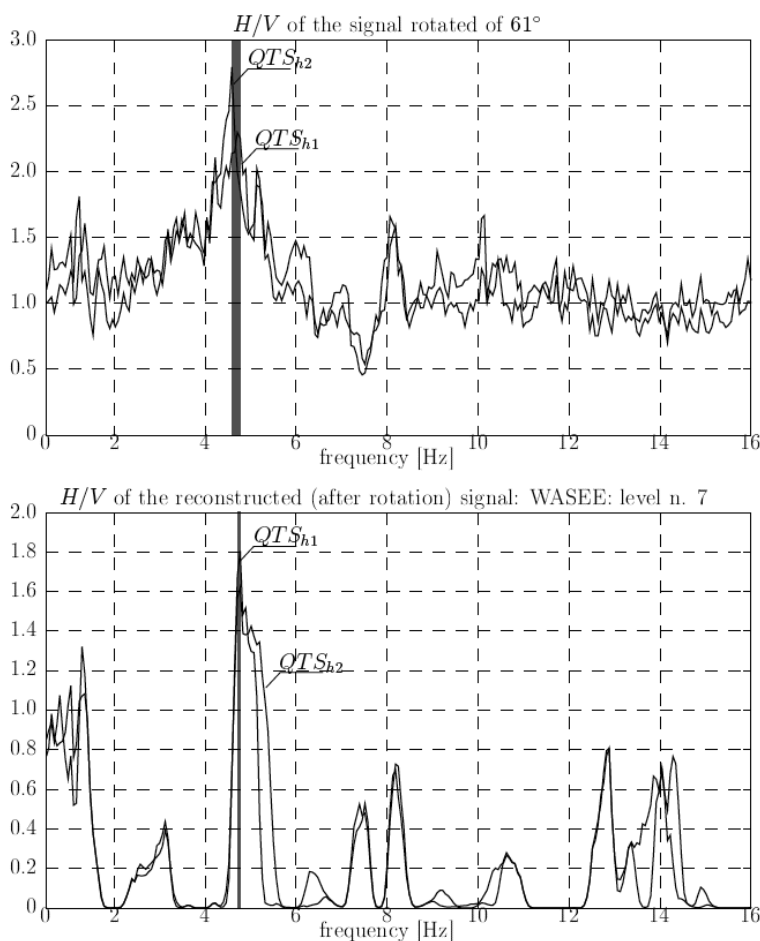


Fig. 13 - Standard H/V ratio and WASEE ratio for the K11 station, after the rotation of the coordinate system by an angle $\theta = 61^\circ$.

direction of noise. For this reason, the application of denoising techniques such as SSA or WASEE may not lead to optimal results. It is, therefore, useful to introduce a rotation of the coordinate system before performing the standard Nakamura (1989) spectral ratio.

In this way, one can determine the directions of maximum and minimum coupling between the QTS 's, which can, in turn, indicate the directions of maximum and minimum isotropy (or minimum and maximum anisotropy). Once the direction of maximum isotropy is known, it is possible to improve the evaluation of the position of the QTS 's peak. This represents what we can call the "isotropic component" of the site effect, which remains significant even under non-ideal conditions of the site and/or the noise sources. It is not always possible, in general, to ascribe the remaining "anisotropic component" to either cause (source or site anisotropy). Of course, in the special case in which the sources can be considered ideal, it is possible to identify local anisotropies. On the other hand, if it is known that the site is homogeneous and has planar layers, probably, the anisotropy would be connected to "local directional noise".

The application of denoising techniques such as SSA and WASEE, after rotation, allows for further improvement with respect to the application of the same techniques on the raw ns and ew

data. The results presented show the potential of a joint application of all three preprocessing techniques.

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