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The fuzzy classification of geometallurgical domains

F. Khorram[1], O. Asghari[1], H. Memarian[1], A.H. Morshedy[2] and X.M. Emery[3]

[1] *Department of Mining Engineering, University of Tehran, Iran*

[2] *Department of Mining and Metallurgical Engineering, University of Yazd, Iran*

[3] *Department of Mining Engineering, Advanced Mining Technology Center, University of Chile, Santiago, Chile*

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ABSTRACT Mineral reserves are heterogeneous by nature, which can lead to a high variability in the intrinsic features of ores and in the responses to mineral processing operations. A geometallurgical model, along with a reliable reservoir block model, helps to quantify the significant variabilities and develop programs to deal with them. It is an important tool for mitigating production risks and improving economic performance in the modern mining industry. In complex porphyry ore deposits, samples with high sulphide copper and high oxide copper will be processed via flotation and heap-leaching units sequentially. Therefore, a geochemical domaining based on total copper grade and its oxide and sulfide fractions can be inferred as the block processing destinations to different processing units or waste dump. Routine classification methods separate two domains with a sharp boundary. Since the error in grade estimation is unavoidable, the geometallurgical classification based on geochemical domaining will face uncertainty. Selecting a range threshold for each boundary, defining fuzzy membership functions, and assigning a membership degree to different classes for each sample would be a solution for this problem. This approach helps remove uncertainty in decision making, reduces the risk and increases the profitability of the project. This geometallurgical model is applied for different block dimensions with the aim of comparing the results.

Key words: block model, geometallurgical modelling, fuzzy classification, porphyry copper.

1. Introduction

Obtained features of mineralogy and geology studies are considered as intrinsic features of the ore body. These features include texture, degree of liberation as well as mineral dimensions which play important role in the behaviour of samples in the mineral processing factory (Coward *et al.*, 2009; Biosvert *et al.*, 2013; Lischuick *et al.*, 2020). Achieving the mentioned behaviour as response features is difficult and expensive, therefore, geometallurgy seeks to find the relationship between the intrinsic and response features and models it through the ore body with the objective of mining operation optimisation and risk reduction (Lamberg *et al.*, 2013; Khorram *et al.*, 2020).

In order to take representative metallurgical samples that determine the natural variability of the processing response, it is necessary to partition a deposit into homogeneous regions in terms of processing properties, called geometallurgical domains (Emery and Ortiz, 2011; Williams, 2013; Rajabinasab and Asghari, 2019). Quantitative rock characteristics such as chemical assay, petrophysical properties, mineralogy, and texture, are used to form these domains with regard to processing properties. Considering the nature of geometallurgical variables and some challenges in working with them, such as non-additivity and measured properties in different scales, selecting a robust simulation and classification method is critical to achieve reliable results (Lund *et al.*, 2015; Deutsch *et al.*, 2016; Dominy and O'Connor, 2016).

Geometallurgical classification based on definite threshold limits can be risky and lead to uncertain results owing to the incompleteness of data and inaccuracy of simulation and classification performance (Demicco and Klir, 2004; Taboada *et al.*, 2006; Ross, 2010). To overcome this problem, fuzzy methods, that are strategies to convert quantitative data to qualitative data, are used (Sinclair and Blackwell, 2002; Vann *et al.*, 2012). In the present paper, a new fuzzy method, which gives the most reliable fuzzy number as thresholds for domaining, is used for the first time in geometallurgical modelling. Given that blocks, as a unit of the processing plant, are important in the efficiency of mine operations, in the last part of the article all of the fuzzy classifications are done for different block sizes for comparison.

2. Geometallurgy and fuzzy approach

Geometallurgy integrates a wide range of mineral processing attributes and intrinsic features of ore and optimises all mining operations based on the maximum available information (Dominy and AusIMM, 2011; Emery and Ortiz, 2012; Lischuick *et al.*, 2020). The design of ore processing stages, optimisation of mining equipment size according to the limitations and operating costs, forecasting and optimisation of mining stages, risk reduction in feasibility, and production and operation stages are affected by geometallurgy (Lamberg *et al.*, 2013). In this paper, the purpose of geometallurgical modelling is to classify the deposit in different zones with their own processing method and to predict the performance of the processing in different parts of the deposit. Table 1 and Fig. 1 show the intrinsic features and their corresponding response properties during processing (Schouwstra *et al.*, 2010; Biosvert *et al.*, 2013; Dominy and O'Connor, 2016).

Table 1 - Intrinsic features and their related tests (Bowell *et al.*, 2011).

Topic	Considered features	Related experiment
Geology	Geological unit and their relation	Mapping
Chemistry	Metal and mineralogical grade	Assay analysis of holes
Mineralogy	Texture, size and other features of minerals	Microscopic images
Physical feature	Hardness and grindability	Bond work index
Processing responses	Recovery	Flotation and hydrometallurgy tests
Geotechnique	Structural features	RQD (Rock Quality Density)

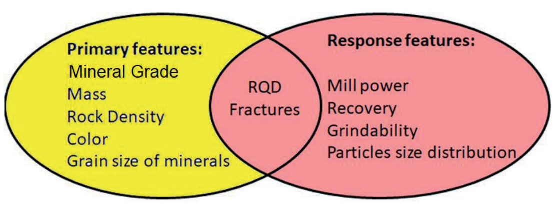

Fig. 1 - Input and its corresponding output characteristics in geometallurgy (Coward *et al.*, 2009).

Ore and waste separation and geometallurgical classification of ore into different units, which reduces the risk of mine planning, is done based on the exact amount of the cut-off grade as a boundary. While error estimation is unavoidable, and considering the data incompleteness, inaccuracy and uncertainty, the fuzzy classification will be helpful in realistic decision-making (Verly, 2005). The fuzzy method is used to make decisions in an environment with insufficient information. For this purpose, instead of separating two classes in a definite boundary, the fuzzy membership function is defined (Taboada *et al.*, 2006; Ross, 2010). Common methods for fuzzifying definitive thresholds are the indicator weighted average (*IWA*) and class membership degree (*CMD*). This method was introduced by Journel (1983) to estimate the spatial distribution in ore and waste zones (Gossage, 1998; Lang *et al.*, 2018). Using the indicators is a strategy to describe the grade of spatial distribution in different thresholds that are used to divide data based on thresholds and to convert quantitative data to qualitative data. Thus, the upper and lower limits of the fuzzy threshold are determined based on the frequency of the data (Vann *et al.*, 2012).

This approach is applied to the data in two ways. The first is before the estimation and use of the point threshold on the input data. The second case is the use of a range threshold on the estimated data. The range of thresholds (membership function) is used to determine the degree of membership of each class (μ). If the number of thresholds is K, the number of indicators is equal to K+1 (Wingle, 1997). The method presented in this article makes it easy to determine the range limits of fuzzy numbers and was used for the first time in geometallurgical classifications (Morshedy *et al.*, 2015). The *IWA* determines the fuzzy indicator of each block based on the threshold range and degree of membership (Fig. 2). This value is calculated for two classes of C_j, C_{j+1} and n degrees of membership for different limits and their corresponding grades based on the following relationship:

$$IWA[G(\mu_i)]_{C_{j+1}}^{C_j} = \frac{\sum_{i=1}^{n} \mu_i \times I[G(\mu_i)]}{\sum_{i=1}^{n} \mu_i}, C_j < IWA[G(\mu_i)]_{C_{j+1}}^{C_j} < C_{j+1} \quad (1)$$

$$CMD = \begin{cases} C_{j+1} - IWA[G(\mu_i)]_{C_{j+1}}^{C_j} & C \in C_j \\ IWA[G(\mu_i)]_{C_{j+1}}^{C_j} - C_j & C \in C_{j+1} \end{cases} \quad (2)$$

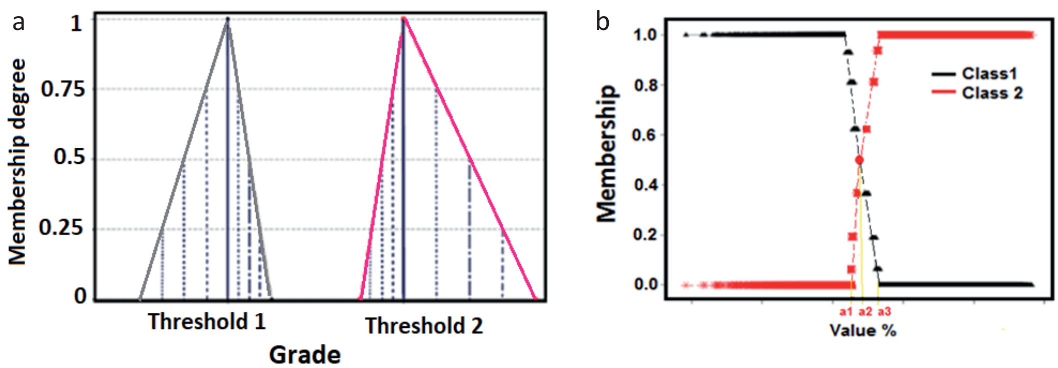

Fig. 2 - Thresholds for *IWA* calculation (a) and membership function of ore and waste classes (b).

3. Case study: the Sungun copper deposit

3.1. Geological description

The Sungun copper deposit is located in the Urmia-Dokhtar volcanic zone of the Alpine-Himalayan belt. Most of Iran's porphyry copper deposits were formed during the magmatic activity of this belt. Previous studies on the geological characteristics of this deposit distinguished three main zones: porphyry copper, skarn, and dikes (Hezarkhani, 2006). Spatial distribution of the "early phase" of hydrothermal activity (hypogene alteration) is more complex than the commonly accepted porphyry copper models. Also, the presence of a "later phase" of hydrothermal activity (supergene alteration) induces a high variability of grade and of oxide/sulfide copper ratio. The complexity of the case study certainly requires to apply a new approach for the geometallurgical model (Shahabpour, 2007; Asghari and Hezarkhani, 2008). Skarn mineralisation exists along the eastern and northern margins of the hydrothermal zone. Intersecting dikes were found mainly in the northern and eastern parts of the Sungun porphyry deposit. Most of these dykes have little or no mineralisation and vary in thickness from a few centimetres to a few metres, which is considered as waste. Along with the Mo, the main copper oxide and sulfide minerals in the Sungun porphyry deposit consist of plagioclase (40-45%), orthoclase (30-35%), amphibole (5-10%), biotite (5-10%), and quartz (5-10%).

Three main rock types have been recognised in the mineralised body and control the copper grade distribution (Hezarkhani, 2006): the Sungun porphyry (SP) stock, skarn mineralisation (SK), and injected dykes (DK). Table 2 demonstrates the statistical parameters of copper grade, for dykes, skarns, Sungun porphyry, and overall.

Table 2 - Statistical parameters of copper grade, for each rock zone and as a total.

	Number of data	Mean	Std. deviation	Skewness	Kurtosis	Minimum	Maximum
DK	9016	0.08	0.22	6.30	59.39	0	3.58
SK	749	0.57	1.28	10.23	154.46	0	23.50
SP	20664	0.53	0.48	2.60	24.02	0	9.61
Total	30430	0.40	0.51	6.23	177.63	0	23.50

The case study deposit is divided into the optimal number of domains, and an area containing the supergene, hypogene, and leach zones is selected (based on stationary and continuity) as the study area (Journel and Huijbregts, 1978). This zone, with the coordinates of origin: X = 8000 m, Y = 4400 m, Z = 1900 m, is shown in Fig. 3. It covers a volume of approximately 0.8 km (longitude), 0.6 km (latitude), and 0.3 km (below the surface). Fig. 3 shows the plan of the grade block model of a part of the Sungun copper deposit, the case study area is located in the western part of the map. The margins of the porphyry intrusion are skarn that is shown in brown, while the borders between the leach and supergen zones, and supergen and hypogene zones are illustrated with red and blue lines. The following data have been used in this research: 1) percentage of Cu, CuO, and Mo assays in drillholes; 2) percentage of Cu and CuO assays in blast holes related to a 6-year period.

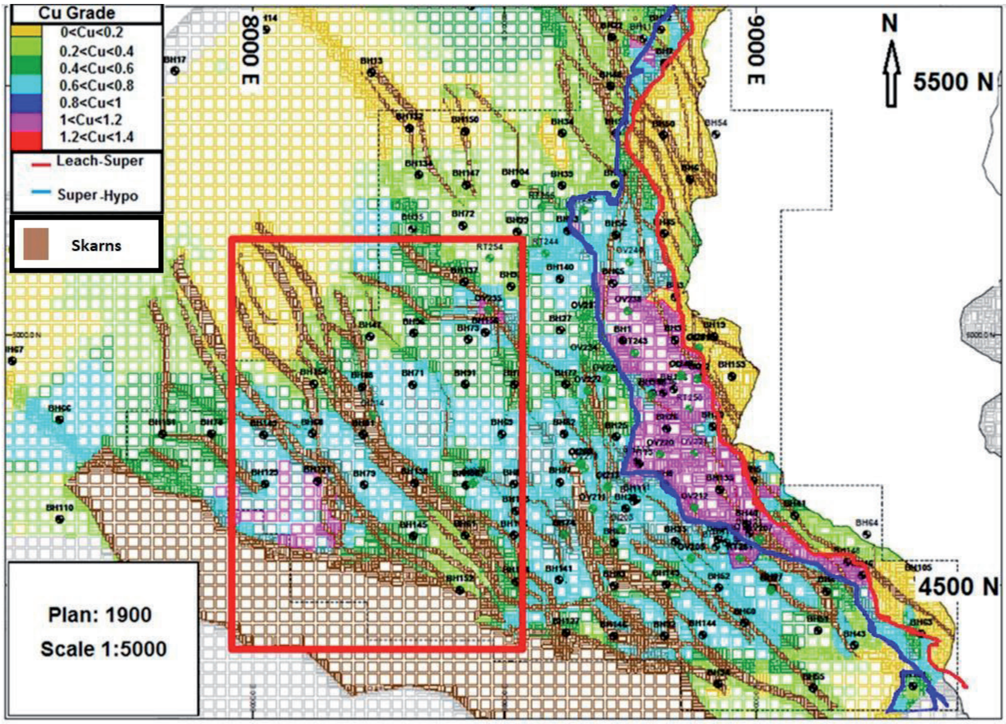

Fig. 3 - Plan of the block model of copper grade in the study area, height 1900 m (Pars Olang Eng. Consultant Co., 2006).

3.2. Geological maps and reports of the study area

Fig. 4 illustrates the location map of drillholes and blast holes in the case study area. Cu and CuO analyses exist in all drillholes, the average distance of adjacent wells is 70-100 m. All assays consists of 2-m down-hole lengths. Furthermore, 4,591 blast-hole samples are used for validation of results. The total copper grade is analysed in all the samples.

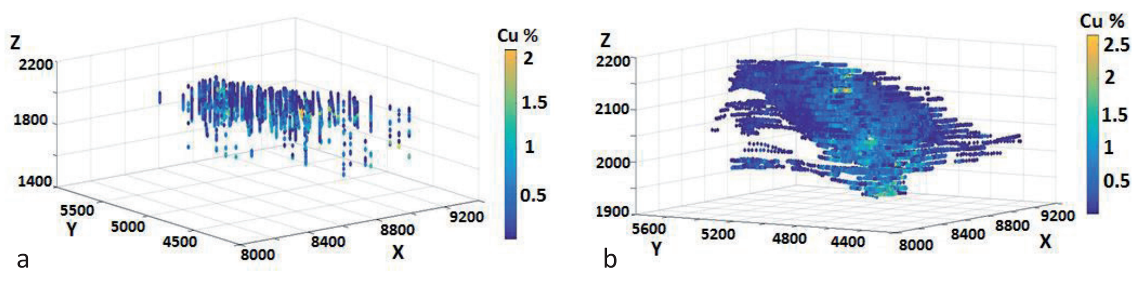

Fig. 4 - Location map and value of available data for Cu grade: a) drillholes; b) blast holes.

The first step to conduct numerical studies is statistical pre-processing. Statistical characteristics and histograms of the frequency of data are shown in Table 3 and Fig. 5. The placement of missing data, and outlier and censored data removal were done. Declustering was used to remove the possible effect of irregular drilling.

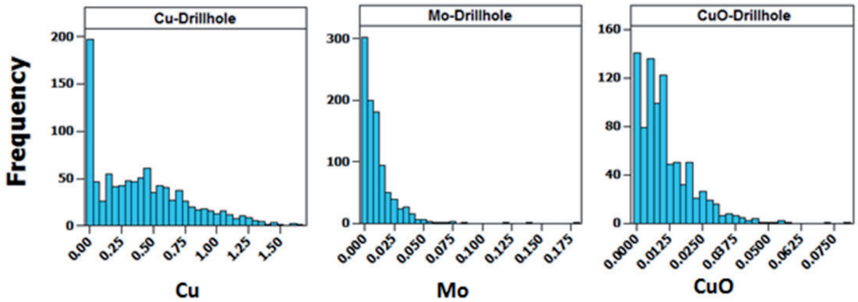

Fig. 5 - Histogram of the frequency of geochemical analysis in drillholes.

Table 3 - Statistical parameters of data.

		Number	Mean	Skewness	Standard deviation
CuO	Blast hole	19491	0.041	7.30	0.079
	Drillhole	1026	0.035	7.39	0.001
Cu	Blast hole	19491	0.330	2.04	0.390
	Drillhole	1026	0.410	0.79	0.350
Mo	Drillhole	1026	0.010	1.85	0.010

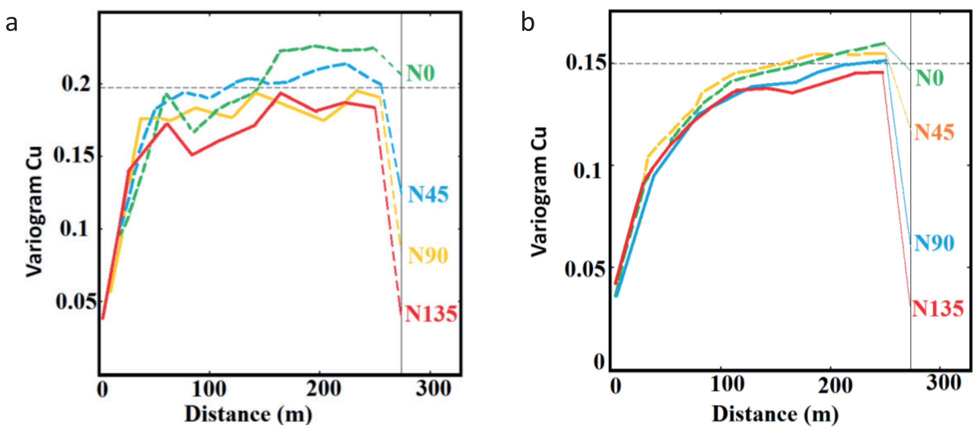

Fig. 6 - Directional variograms for Cu of blast hole (a) and drillhole (b).

Table 4 - Anisotropic ellipsoid parameters

	Major axis/medium axis	Major axis/minor axis	Azimuth	Dip	Rake
Cu Drillhole	1.5	1.6	0	45	40
Cu Blast hole	1.1	1.2	135	90	0

After preliminary statistical studies, variography of Cu in drillholes and blast holes was done in different directions for three-dimensional structural and spatial analysis of the considered data. The fitted variogram model and also the attributes of the anisotropic ellipsoid are shown in Fig. 6 and Table 4.

4. Proposed algorithm for fuzzy geometallurgical modelling

The fuzzy triangular number is used in this paper because of its simplicity and generalisability. In the proposed method for determining the features of the threshold function, the confusion matrix and point threshold is used. To this end, we perform the mentioned classification based on a threshold around the cut-off grade, and the grade that reaches the highest correct classification rate (*CCR*) will be taken as a_2 or the vertex of the triangular function. The a_1 is the mean of the false negatives or underestimated blocks (ore blocks, which are classified as waste), a_3 is the mean of the false positive or overestimated blocks (waste blocks, which are classified as ore) (Moon *et al.*, 2009; Zapata *et al.*, 2010). The mean blast-hole values of copper and copper oxide in classified blocks are used as real data. The triangular fuzzy threshold is introduced by the triangle whose vertices provide the most confidence in the classification (Fig. 7). The CCR is defined as the ratio of the number of samples in the confusion matrix diameter to the total number of samples with a value between 0 and 1 (Verly, 2005; Han *et al.*, 2006). In this equation the true positive (*TP*), true negative (*TN*), the false positive (*FP*), and false negative (*FN*) are used:

$$CCR = \frac{TP+TN}{FP+TP+FN+TN} \tag{3}$$

This value is very close to the definite cut-off grade on which the indicator of ore and waste are obtained based on:

$$I(Value) = \begin{cases} 0 & Value < \mathrm{Zc} \\ 1 & Value > \mathrm{Zc} \end{cases} \tag{4}$$

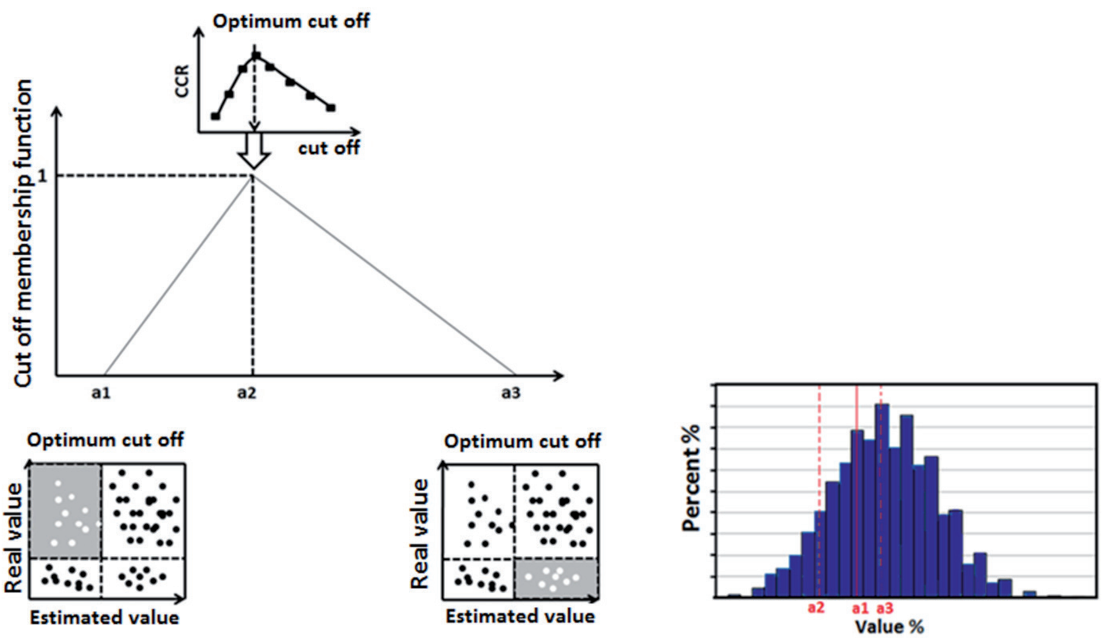

Fig. 7 - Determining the triangular fuzzy number base on correctness coefficient rate.

The class membership function in the introduced method is linear and has 0 membership for the ore class up to a_1 grade. This function ranges from a_1 to a_2, which gives the highest *CCR* in the estimated data, with a fixed line slope of 0.5 degrees. This function, from a_2 reaches to a_3 and membership level of 1 with a steady line, a_3 is the overestimation and the average value of the waste that has been classified as ore thus provides a reliable interval to separate ore and waste.

$$\mu_T(Z) = \begin{cases} 0 & Z < a_1 \\ \dfrac{Z-a_1}{a_2-a_1} & a_1 < Z < a_2 \\ \dfrac{a_3-Z}{a_3-a_2} & a_2 < Z < a_3 \\ 0 & Z > a_3 \end{cases} \tag{5}$$

$$\mu_{Class}(Z) = \begin{cases} 0 & Z < a_1 \\ \dfrac{Z-a_1}{2(a_2-a_1)} & a_1 < Z < a_2 \\ \dfrac{Z+a_3-2a_2}{2(a_3-a_2)} & a_2 < Z < a_3 \\ 1 & Z > a_3 \end{cases} \tag{6}$$

In this algorithm, the block simulation model of total and oxide copper is used for the fuzzy classification of geometallurgical domains. It is assumed that each mined block has three possible destinations: a) flotation if the total copper is above 0.15 (ore-waste cut-off threshold) and the dissolution ratio of CuS/Cu_{total} is less than 0.9 (the threshold for processing destination); b) waste dump, if the total copper is less than 0.15, and if the block achieves other conditions than these two conditions, it will be transferred to heap leaching. So we will have ($CuS/Cu_{tot} = Sol$):

$$\mathrm{I}(Cu) = \begin{cases} (\mathrm{Waste})0 & Cu < \%0.15 \\ (\mathrm{Ore})1 & CuO > \%0.15 \end{cases} \tag{7}$$

$$\mathrm{I}\left(\frac{CuS}{Cu_{tot}}\right) = \begin{cases} (\mathrm{Flotation})0 & \dfrac{CuS}{Cu_{tot}} > 0.9 \\ (Heap - \mathrm{Leaching})1 & \dfrac{CuS}{Cu_{tot}} < 0.9 \end{cases} \tag{8}$$

$$\mathrm{I}\left(Cu.\frac{CuS}{Cu_{tot}}\right) = \begin{cases} Cu > 0.15 \\ Cu < 0.15 \\ Sol > 90 \\ Sol < 90 \end{cases} = \begin{cases} Cu > 0.15 & Sol > 90 & \text{Flotation} \\ Cu < 0.15 & Sol > 90 & \\ Cu > 0.15 & Sol < 90 & \text{Heap-leaching} \\ Cu < 0.15 & Sol < 90 & \end{cases} \tag{9}$$

Based on these thresholds and considering that each of them is defined as a triangular function, in total, we have several classes (Fig. 8). The membership function of thresholds (μT), for each class can be determined according to the equation of the line. Since the waste blocks are not transferred to the processing plant, the processing destination of those blocks, which include a high degree of membership in the ore class, will be examined. Here the CuS/Cu_{tot} is called as dissolution ratio (*Sol*).

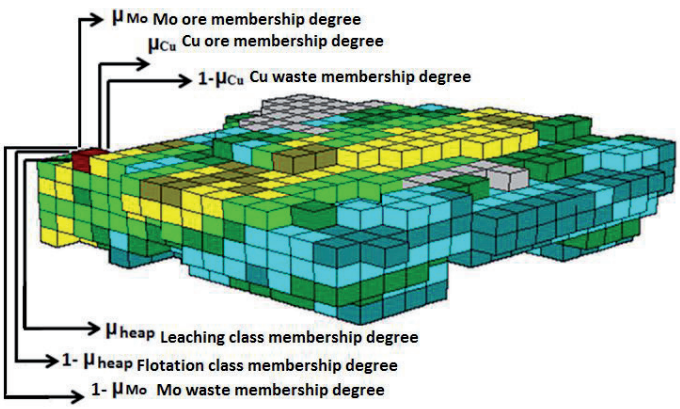

Fig. 8 - Schematic diagram of assigning membership degree to each block.

In order to change the support approach, the membership coefficients of ore and waste for block models with dimensions of 30×30×15, 25×25×12.5, and 20×20×10 m^3 and ore value for each of them are calculated. The function of the ore value is examined in order to convert the grade to the relative value of the economic elements. This function determines the total value of each block according to the influential variables. To determine the cumulative value of the ore, the weighted coefficient of the element, the *CMD* μ and the grade of the valuable element are needed. Finally, the value of the ore is calculated by the sum of the multiplication of the weights of each element, the grade and the degree of membership of the ore for the desired element (Soltani and Hezarkhani, 2011).

$$Ore\ Value = w_{Cu} \times \mu_{CuOre} \times g_{Cu}. \tag{10}$$

After calculating the ore value, the false classification model is created for each of the models with different dimensions (Chai *et al.*, 2004). A block with underestimation (*FN*) is an ore block that is misclassified and transferred to the waste dump. In this case, the calculated ore value for the block is lost and is determined as the incorrect classification cost:

$$I(X_{block}, Z_{cCu}) = 1 \quad , \mu(X_{Ore}) \leq 0.5 (FN). \tag{11}$$

False positive (*FP*) is also a waste that is misestimated as ore, and increase ore tonnage, and all extractive, operating, and processing costs per ton are incorrectly calculated for that block:

$$I(X_{block}, Z_{cCu}) = 0 \quad , \mu(X_{Ore}) \geq 0.5 \ (FP). \tag{12}$$

It is worth noting that the disadvantage of ore loss is greater than the additional costs of extraction (Rogers and Kanchibotla, 2013). Fig. 9 shows the flowchart of the proposed method.

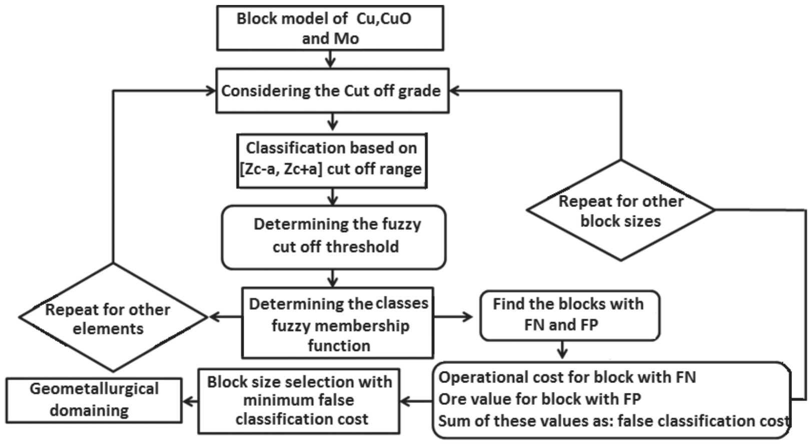

Fig. 9 - Flowchart of proposed method steps; *FP* and *FN* are false positive and negative.

5. Ore-waste classification

In order to implement the proposed algorithm on the study area, the average of 50 values of multivariate simulation of copper and copper oxide in block models with the considered dimensions, have been calculated. For this purpose, the DBSIM method was applied to the independent factors.

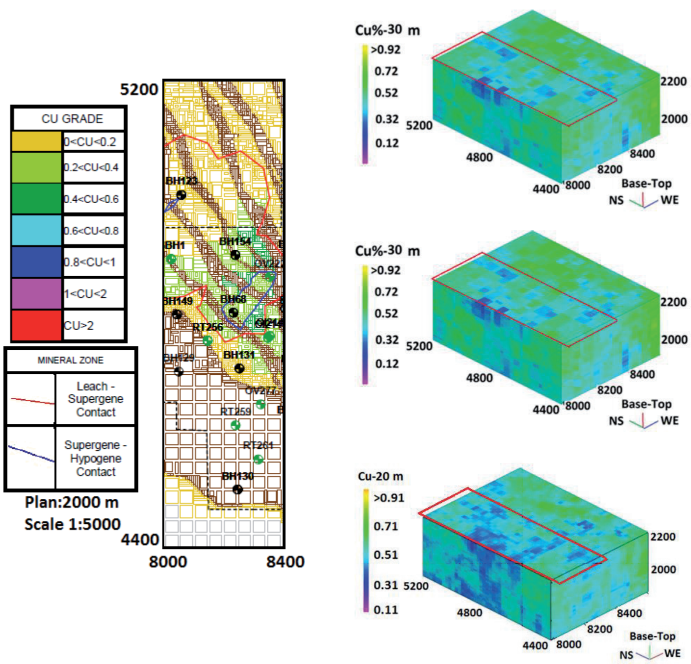

Fig. 10 - The estimated copper and geological map of a part of the case study area.

The real cut-off grade for Cu is considered 0.15 for the actual data (average of the actual values in the equivalent blocks) and a range of the cut-off grade [0.1-0.2] is considered for the estimated 30×30×15 m^3 blocks. The real, and range of cut-off for Mo is about 0.006 and [0, 0.1], the correctness classification rate of ore and waste was calculated on the estimation models and based on each value of their range. The grade in which this value is maximised is chosen for the vertex of the fuzzy triangular number, which is 0.15. Also (a_1, a_3) is equal to (0.09, 0.25) based on the average values that have been overestimated and underestimated in these classifications (Figs. 11 and 12).

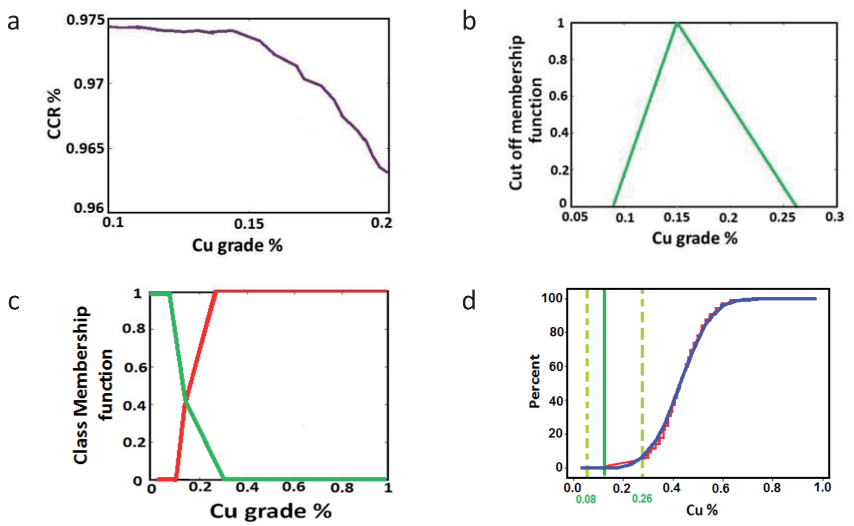

Fig. 11 - a) *CCR* for different cut-off; b) Cu cut-off membership function; c) ore and waste membership function; d) relation of *CDF* with limits of fuzzy cut-off 30×30×15 m^3.

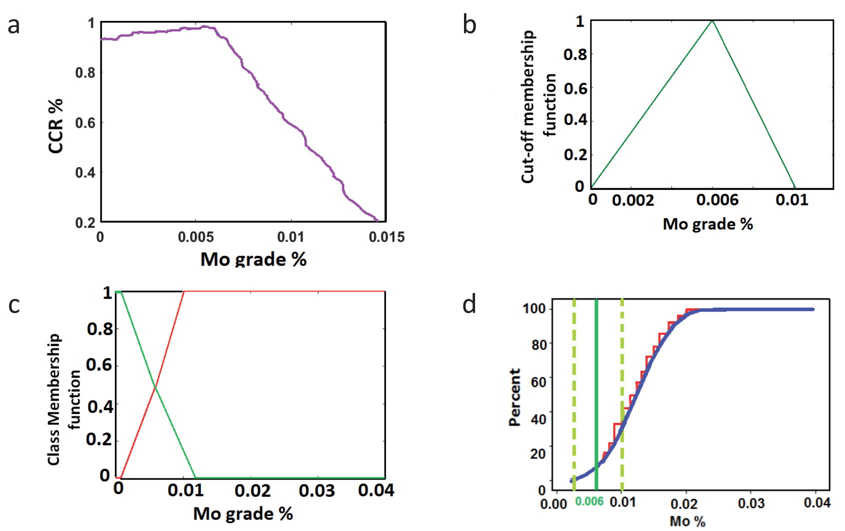

Fig. 12 - a) *CCR* for different cut-off; b) Mo cut-off membership function; c) ore and waste membership function; d) relation of *CDF* with limits of fuzzy cut-off, 30×30×15 m^3.

6. Misclassifications based on copper

As explained before, the ore value function is examined in order to convert the grade to the relative value of the economic elements. This function determines the total value of each block according to the price of copper and molybdenum. After calculating the value of the ore, the misclassification model for different block dimensions is formed. For a block with a false negative, the calculated ore value for the block (based on copper and molybdenum) will be lost. For the block with false positive, the extraction, operating and processing costs per ton are incorrectly calculated. The sum of all these values is introduced as the incorrect classification cost for a block model. Fig. 13 shows the spatial model of ore and waste membership degree in the 30×30×15 m^3 blocks. Based on the identified areas in the figures, which are considered to be waste with a definite threshold, it is possible to visually observe the correctness of the fuzzy classification and decision-making band around each class. It can also be inferenced that the modelled blocks as waste correspond to the high concentration of the dykes (which has low grades of Cu and Mo) (Fig. 10).

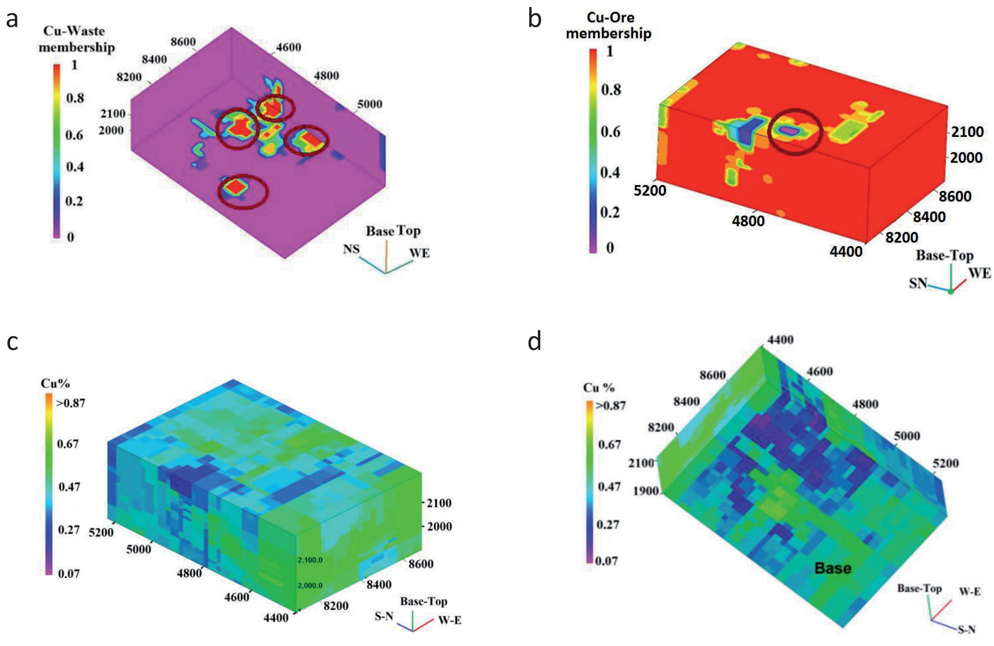

Fig. 13 - a, b) Spatial model of Ore/Waste membership degree; c, d) spatial model of Cu grade.

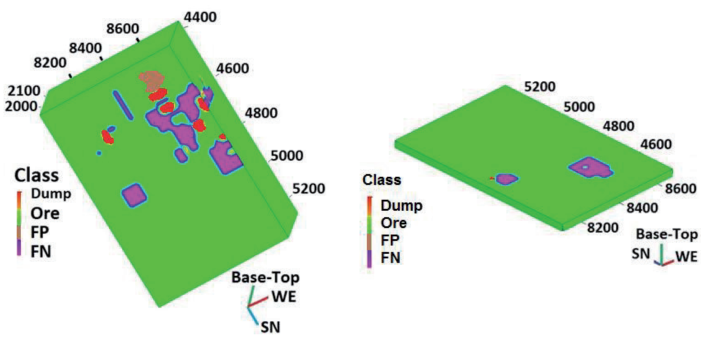

Fig. 14 - Spatial model of classification groups 30×30×15 m^3, *FN* and *FP* are false negative and false positive.

Fig. 14 shows the different classification groups based on the cut-off grade for the 30×30×15 m^3 block. As can be seen from the figures, at higher altitudes, less misclassification occurs, which can be due to the high density of the specimens on the surface.

7. Misclassifications based on copper and molybdenum

According to our case study, which is copper and molybdenum mining, and based on the fact that molybdenum processing in Sungun copper mining is done in continuation of copper processing, in this section, to determine the false classification and the resulting cost, the net smelter return (*NSR*) will be calculated. All economic values used in these calculations are from reputable sources related to 2004 and only to create the correct ratio of these two elements (Rendue, 2013). Copper and molybdenum cut off grades are considered 0.15 and 0.01 according to the factory reports, and the specific gravity of these metals is considered 8.9 and 10.2 g/cm^3, respectively.

In the mentioned relationships, Indicators 1 and 2 refer to copper and molybdenum metals, respectively. The required parameters for the calculations are given in Table 5, the relationship between the *NSR*, X_1 and X_2 thresholds is shown in Fig. 15. *NSR* corresponds to a metric ton of material with an average grade of X and is calculated as follows (Rendue, 2013):

$$NSR\ (x_1, x_2) = x_1 r_1 p_1 (V_1 - R_1) + x_2 r_2 p_2 (V_2 - R_2) - (Cs + Ct)/K = 2.149x_1 + 7.39x_2 - 2.014 \qquad (13)$$

$$NSRc = (Po_1 + Po_2 - P_w) + (O_o - O_w) + (M_o - M_w) = 3.55 \qquad (14)$$

with *NSRc* in \$/t.

Table 5 - Mineral processing and mining operational cost (Rendue, 2013).

r_1	=	89% copper flotation plant recovery
p_1	=	96.5% copper smelting recovery
r_2	=	61% molybdenum flotation plant recovery
p_2	=	99% molybdenum roasting recovery
V_1	=	\$1.20 value of one pound of copper sold
V_2	=	\$6.50 value of one pound of molybdenum sold
R_1	=	\$0.065 refining cost per pound of copper
K	=	72 metric tons of ore must be processed to produce one metric ton of concentrate
Cs	+	Ct = \$145 smelting and freight costs per metric ton of concentrate
R_2	=	\$0.95 conversion, roasting, and freight costs per pound of molybdenum
Mo	=	\$1.00 mining cost per metric ton of ore milled
Po_1	=	\$3.00 mill processing cost per metric ton milled
Po_2	=	\$0.15 incremental molybdenum processing cost per metric ton milled
Oo	=	\$0.50 overhead cost per metric ton milled
Mw	=	\$1.00 mining cost per metric ton wasted
Pw	=	\$0.05 processing cost per metric ton wasted
Ow	=	\$0.05 overhead cost per metric ton wasted

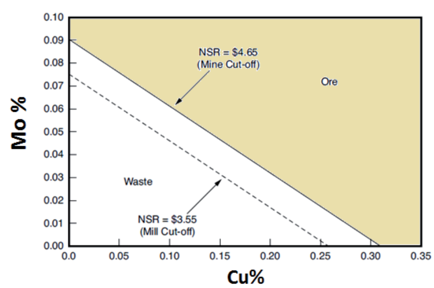

Fig. 15 - The relation of *NSR* and Mo&Cu (Rendue, 2013).

To determine the cumulative ore value, the weighted coefficient of the element, the *CMD* μ of the valuable and considered elements are determined:

$$\text{Ore Value} = w_{Cu} \times \mu_{CuOre} \times g_{Cu} + w_{Mo} \times \mu_{MoOre} \times g_{Mo} \tag{15}$$

$$\mathrm{I}(NSR) = \begin{cases} (Mo\&Cu\ Waste)0 & \mathrm{NSR} < 3.55 \\ (Mo\&Cu\ Ore)1 & \mathrm{NSR} > 3.55 \end{cases} \tag{16}$$

$$I(NSR_{block}, NSR_c) = 1 \quad , \mu(X_{CuOre}) + \mu(X_{MoOre}) \leq 1(FN) \tag{17}$$

$$I(NSR_{block}, NSR_c) = 0 \quad , \mu(X_{CuOre}) + \mu(X_{MoOre}) \geq 1(FP) \tag{18}$$

$$\mathrm{I}(Mo) = \begin{cases} (Mo\ Waste)0 & \mathrm{Mo} < 0.01 \\ (Mo\ Ore)1 & \mathrm{Mo} > 0.01\,. \end{cases} \tag{19}$$

The fuzzy membership degree of each block related to Mo class (greater than 0.01% cut off grade), ore value (Eq. 18), and the cost of misclassification are calculated based on the *NSR* for the model with dimensions of 30×30×15 m^3. For this purpose, all extraction, operation and processing costs that are mistakenly spent for the FP group are calculated for both copper and molybdenum elements and according to Table 5.

Also, the cost of misclassification based on copper cut-off grade, copper costs, and price for each model with dimensions of 30×30×15, 25×25×12.5, and 20×20×10 m^3 is calculated. The values of the fuzzy number (a_1, a_2, a_3) in order to determine the degree of fuzzy membership of ore and waste for block models with medium and small dimensions are equal to (0.22, 0.16, 0.10) and (0.22, 0.14, 0.06). The results of the calculation of the misclassification cost are shown in Table 6.

Table 6 - The results of false ore/waste classification cost.

	Misclassification cost	Loss of operational cost	Loss of ore value	Cut off unit
30×30×15 m^3	2313.8	228.26	2085.43	NSR ($/ton)
	9.3×10^9	3.8×10^8	8.9×10^9	Cu($)
25×25×12.5 m^3	2.4×10^{10}	1×10^{10}	1.4×10^{10}	Cu($)
20×20×10 m^3	9×10^9	3.5×10^8	8.6×10^9	Cu($)

As can be seen in Table 6, the cost of misclassification for the block model with dimensions of 20×20×10 m^3 has the lowest value. The use of these dimensions in grade modelling reduces the risk of misclassification of ore and waste, is economical, and results in lower costs in the extraction and processing stages.

8. Processing destinations

Since the cost of heap leaching and flotation operations are different, forecasting and modelling the blocks that are transported in each processing path are effective in the economic planning of the mine. Therefore, based on the estimated *CuO/Cu* threshold for each block as an intrinsic feature and the correspondence between this feature and the block destination, fuzzy modelling was performed. For this purpose, the simulated *CuO/Cu* ratio of each block is classified according to the definitive threshold of 0.09 (cut-off) and against this real value (*CuO/Cu* average of blast holes in a block). The *CCR* of simulation blocks with flotation destination *CuO/Cu* < 0.9 and heap leaching *CuO/Cu* > 0.9 were calculated based on the threshold range. The vertex of the triangular fuzzy function is 0.95 and its lower and upper limits are 0.08 and 0.12 (Fig. 16). Table 7 shows the average grade of copper and copper oxide in each of the geometallurgical domains. Accordingly, the average total copper in the blocks with the heap leaching destination is less than this amount in blocks with the destination of flotation, and the average copper oxide in these blocks is more than the floating blocks.

Table 7 - Mean grade of CuO and Cu of different domains.

Destination	CuO	Cu
Dump	0.005	0.028
Flotation	0.056	0.668
Leaching	0.157	0.184

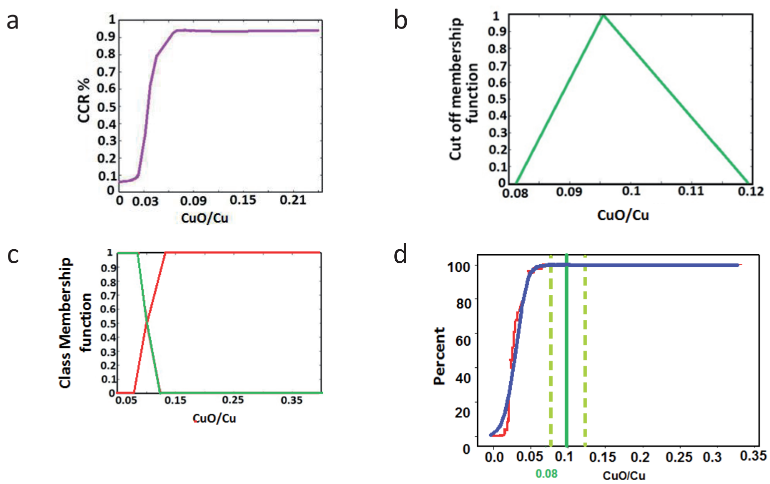

Fig. 16 - a) *CCR* for different cut-off; b) *CuO/Cu* cut-off membership function; c) ore and waste membership function; d) relation of *CDF* with limits of fuzzy cut-off, 30×30×15 m^3.

The membership coefficient of each block to a geometallurgical domain can be determined based on the simulated values of that block. Figs. 17 and 18 show the spatial model of the membership coefficients of the classes with the destination of flotation and heap leaching. Based on these figures, the blocks classified with the heap-leaching destination correspond to the high copper oxide grade sections and also have an acceptable fit with the high concentration of dykes. Also, the middle of the eastern part and the south-eastern part of the model have copper

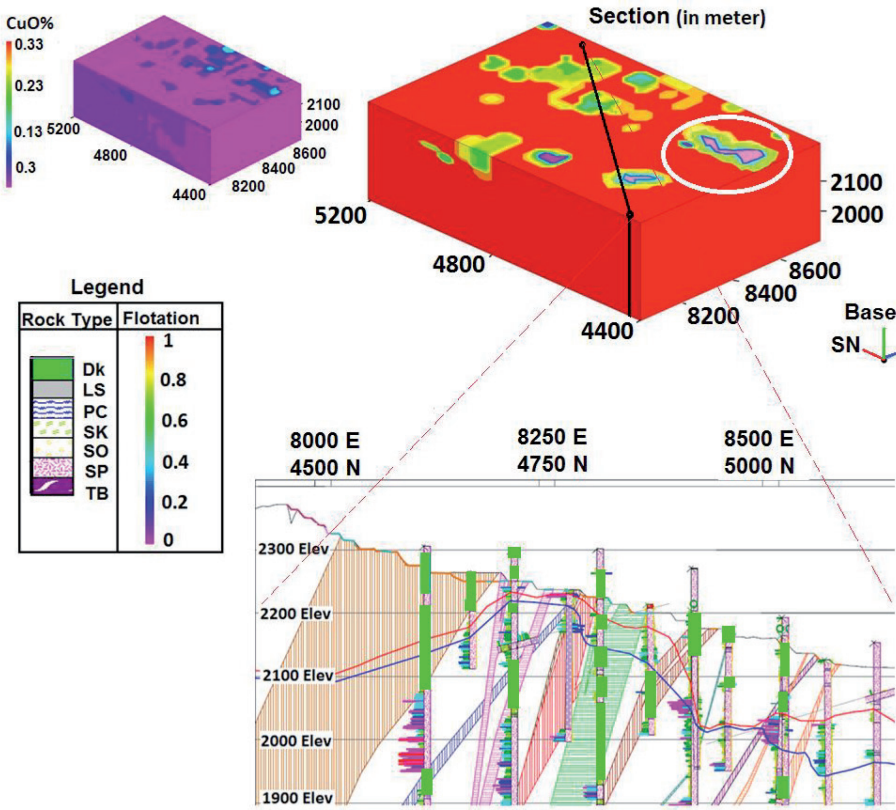

Fig. 17 - Spatial model of flotation class membership, CuO grade of 30×30×15 m^3 and geological map of considered section.

grade higher than 0.6 and are assumed to be transferred to the flotation circuit. However, due to the high grade of copper oxide in these areas, the processing destination of the blocks changes to heap leaching. Also in levels that are close to the leached zone, the heap leaching class is more extensive. The north-eastern sections of the model correspond to the high concentration of the dykes; the reason for which could be the accumulation of more oxide minerals at the site of fluid flow and dyke formation.

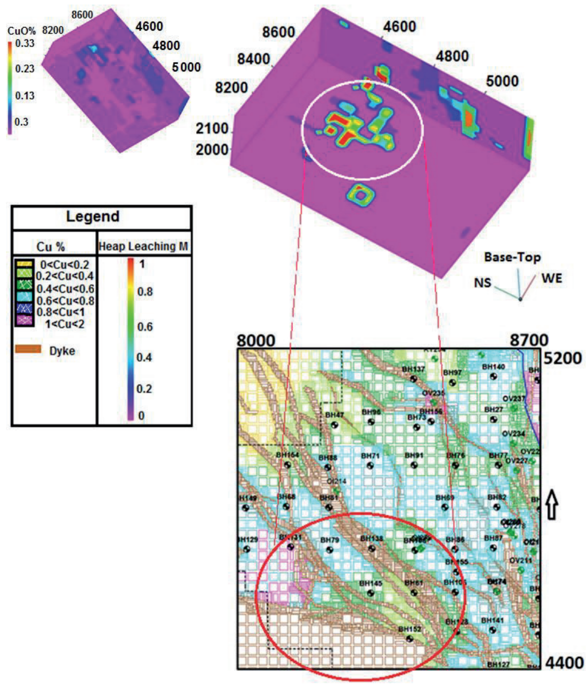

Fig. 18 - Spatial model of heap leaching class membership, CuO grade of 30×30×15 m^3 and geological map.

9. Conclusions

Geometallurgy modelling by including metallurgical properties in reservoir models provides the possibility of realistic mine designing and optimising. The ultimate goal of geometallurgical modelling is to design a reservoir model in which each block includes a prediction of probable destination and behaviour in the processing plant, in order to decrease the decision uncertainty of the subsequent mining operations. Estimation and simulation of ore intrinsic features comes with an inevitable error, and this error is one of the reasons for the modelling uncertainty. In this article, the copper and copper oxide grades were simulated, and the blocks with waste stockpile, flotation (as ore) and heap leaching (as ore) probable destinations were classified with the proposed fuzzy algorithm. Through the fuzzy logic-based method, every block includes the fuzzy membership degree for each geometallurgical class (or block destinations). In this situation, decision making will be accompanied by lesser uncertainty and the risk of the next operations will be diminished.

The results of the method implementation on the study area have an acceptable agreement with the actual data of the mine, and the accuracy of the model is confirmed by them.

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Corresponding author: Farzaneh Khorram
Department of Mining Engineering, University of Tehran
North Kargar, Tehran, Iran
Phone: +98 9021364413, e-mail: farzanehkhorram@ut.ac.ir